CyFlex® Knowledge Article

Crank Angle Sampled Moving Average - Calculations

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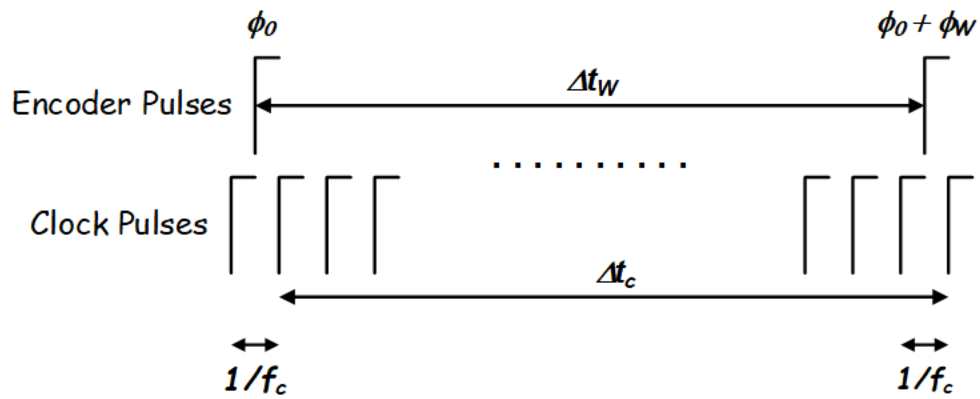
CASMA Angular Velocity Filtering

The torque produced by an engine as a function of crank angle is very dynamic due to the compression and expansion cycle that occurs in each individual cylinder. Cylinder to cylinder variations may also occur that lead to additional torque variations. Variations in torque inevitably produce variations in angular crank velocity. For engine performance development work, the focus is primarily on the average angular velocity and the effect of individual cylinder events is considered to be noise that we would like to filter out.

For a four-stroke cycle engine, one complete cycle of every cylinder will occur every two crank revolutions. It is desirable to set up a Crank Angle Sampled Moving Average (CASMA) filter to smooth out the speed variations over one complete cycle. Other windows are also useful at times. The most common would be one revolution and 720° / n where n is the number of cylinders.



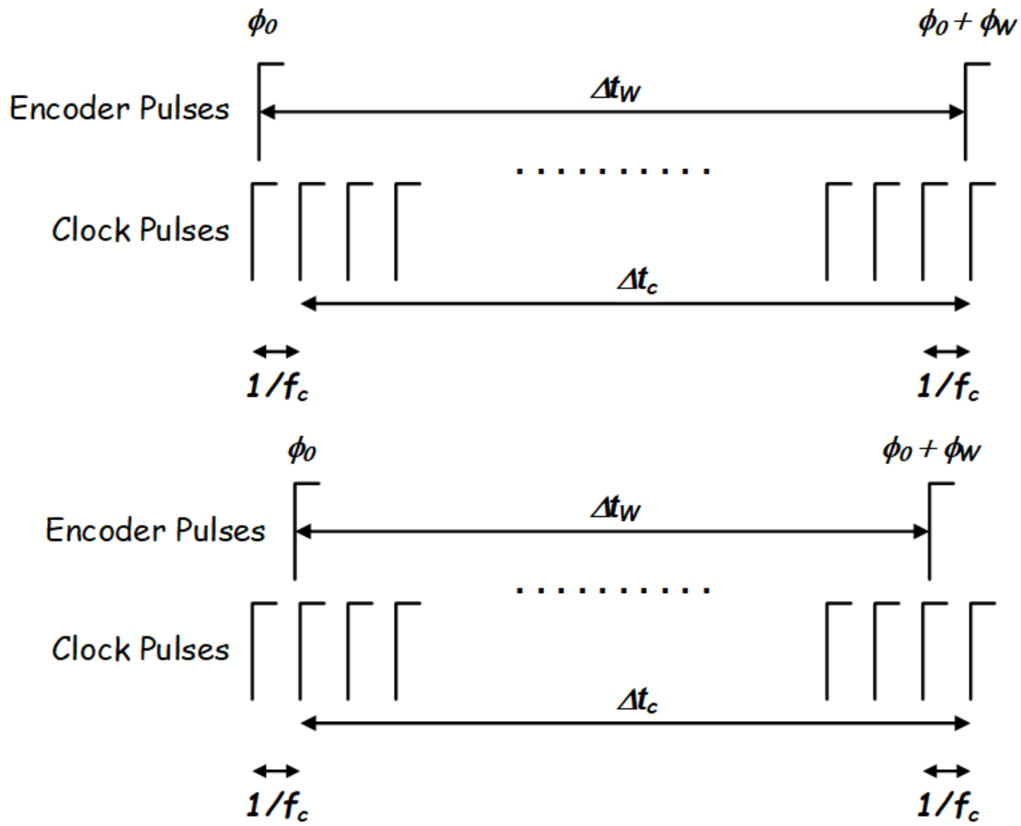
Given the input from a shaft encoder, we can determine the actual angular velocity, ω_{act}, over some crank angle window, ϕ_W, if we can measure the time, Δt_W, between the appropriate number of encoder pulses. The average angular velocity over that crank angle window can then be calculated as

$$\omega_{act} = \frac{\phi_W}{\Delta t_W} \qquad \text{casma.1}$$

We attempt to measure the time between encoder pulses by counting the number of pulses from a clock with frequency f_c to determine the time, Δt_c, between the clock pulses that occur immediately after appropriate encoder pulses. The calculated angular velocity, ω_c, is then:

$$\omega_c = \frac{\phi_W}{\Delta t_c} \qquad \text{casma.2}$$

The number of clock pulses we count will be the same for either of the two extreme cases illustrated below. But we can be confident that our measurement will be off by no more than the period of one clock pulse or $\frac{1}{f_c}$.



So the worst-case scenario is that:

$$\Delta t_c = \Delta t_W \pm \frac{1}{f_c} \qquad \text{casma.3}$$

Solving casma.1 for Δt_W and substituting into casma.3 gives:

$$\Delta t_c = \frac{\phi_W}{\omega_{act}} \pm \frac{1}{f_c} \qquad \text{casma.4}$$

So the largest error we would expect in our angular velocity calculation would be given by:

$$\omega_{act} - \omega_c = \omega_{act} - \frac{\phi_W}{\Delta t_c}$$

$$= \omega_{act} - \frac{\phi_W}{\frac{\phi_W}{\omega_{act}} \pm \frac{1}{f_c}}$$

$$= \omega_{act} - \frac{\phi_W f_c \omega_{act}}{\phi_W f_c \pm \omega_{act}} \qquad \text{casma.5}$$

$$= \frac{\phi_W f_c \omega_{act} - \phi_W f_c \omega_{act} \pm \omega_{act}^2}{\phi_W f_c \pm \omega_{act}}$$

$$= \frac{\pm \omega_{act}^2}{\phi_W f_c \pm \omega_{act}}$$

For clock frequencies in the megahertz range and any reasonable crank angle window, we can show that:

$$\omega_{act} << \phi_W f_c \qquad \text{casma.6}$$

So for all practical purposes the magnitude of the worst-case resolution would be:

$$\left|\omega_{act} - \omega_c\right| \le \frac{\omega_{act}^2}{\phi_W f_c} \qquad \text{casma.7}$$

As an example, if we assume an angular velocity of 2500 rpm, a window of 720°, and a clock frequency of 40 MHz, the maximum expected error would be 0.0013 rpm.

If we only calculate the angular velocity one filter window at a time, the update rate would be unacceptably low at low angular velocities. It is therefore necessary to sample over some smaller crank window, ϕ_S . The calculations are still relatively straightforward, but now involve managing a FIFO buffer of $\frac{\phi_W}{\phi_S}$ subintervals. The sampling frequency, f_S , is given by:

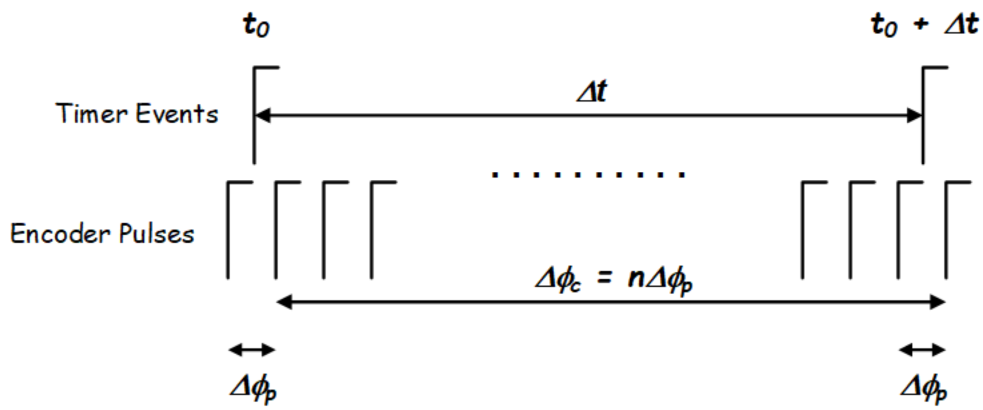
$$f_S = \frac{\omega_{act}}{\phi_S} \qquad \text{casma.8}$$

The choice of sample window width is a balancing act between the capability of the processor performing the CASMA calculations at high angular velocities up to 2500 rpm for large engines and the desire to have updates at a minimum of 100 Hz at an idle speed of 500 rpm. A 30-degree sample window would give the required minimum rate at idle. A smaller window is desirable since we would like to acquire meaningful data during engine cranking.

Special provisions need to be made to capture data during the engine starting process. The entire data window will not contain meaningful data as the engine first starts to rotate. The engine angular velocity should be calculated based on the available data set which will continue to grow until the specified window length is reached.

Instantaneous Angular Velocity

It is also desirable to calculate an "instantaneous" angular velocity based on data acquired over a period that corresponds to the rate at which data is transmitted to the data acquisition system. The simplest way of doing this is often called a "frequency-based measurement" and is illustrated below.



The number of shaft encoder pulses, *n*, are counted between successive timer events that occur at a frequency that corresponds to the data transmission rate. The angular velocity is calculated as:

$$\omega_c = \frac{\Delta\phi_c}{\Delta t} = \frac{n\Delta\phi_p}{\Delta t} \qquad \text{casma.9}$$

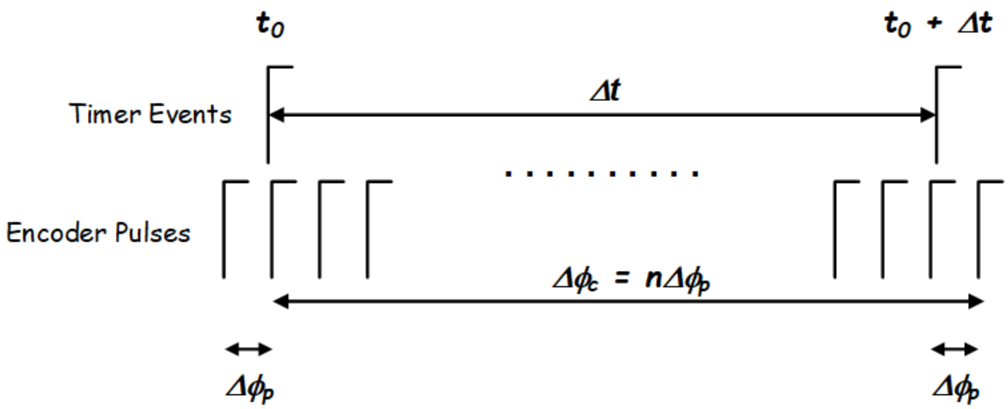
For a shaft encoder having N pulses per unit of angular displacement, the angle between pulses, $\Delta\phi_p$, is given by:

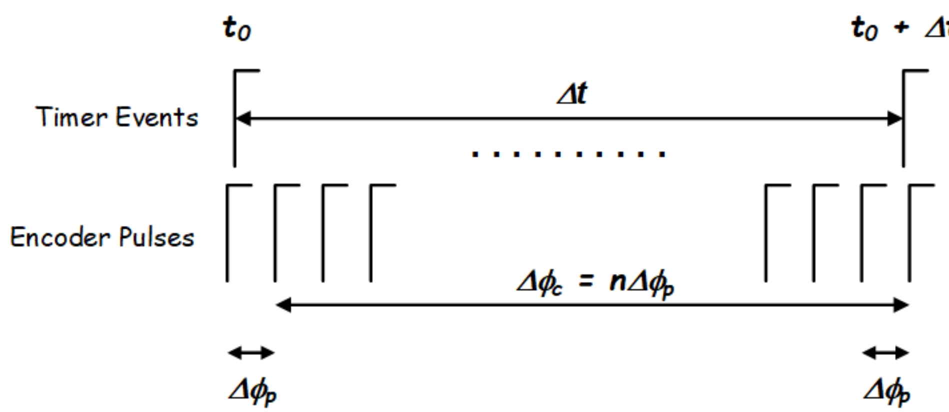
$$\Delta\phi_p = \frac{1}{N} \qquad \text{casma.10}$$

So

$$\omega_c = \frac{n}{N\Delta t} \qquad \text{casma.11}$$

The number of encoder pulses we count will be the same for either of the two extreme cases illustrated below. Which means that our crank angle measurement may be off by as much as the angle for one encoder pulse or $\Delta\phi_p$ from the actual value $\Delta\phi_{act}$.





So

$$\begin{aligned} \Delta\phi_{act} &= \Delta\phi_c \pm \Delta\phi_p \\ &= n\Delta\phi_p \pm \Delta\phi_p \\ &= (n \pm 1)\Delta\phi_p \\ &= \frac{(n \pm 1)}{N} \end{aligned} \qquad \text{casma.12}$$

and the largest error we would expect in our angular velocity calculation would be given by:

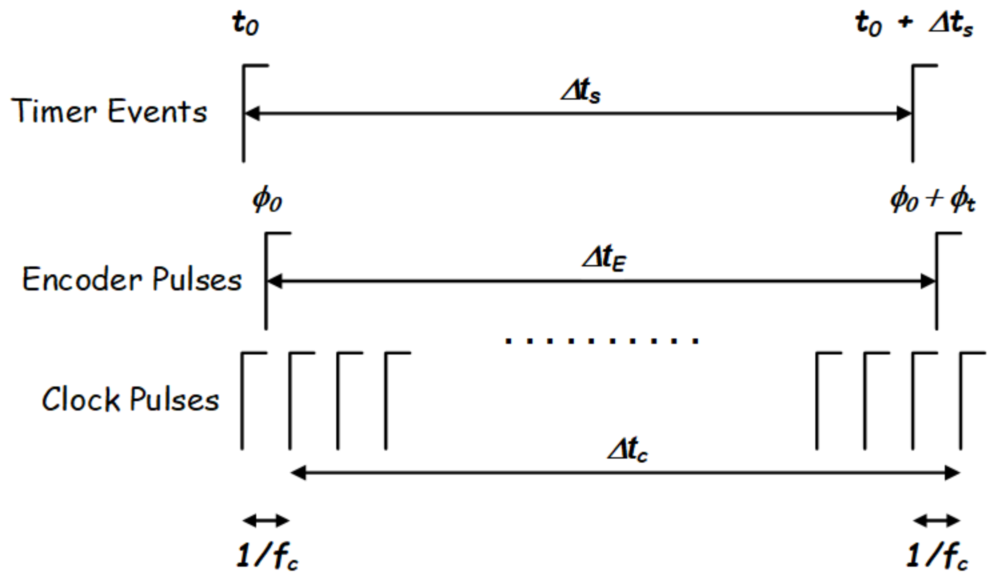
$$\begin{aligned} \omega_{act} - \omega_c &= \frac{\Delta\varphi_{act}}{\Delta t} - \frac{\Delta\varphi_c}{\Delta t} \\ &= \frac{n \pm 1}{N\Delta t} - \frac{n}{N\Delta t} \\ &= \frac{\pm 1}{N\Delta t} \end{aligned} \qquad \text{casma.13}$$

Or in terms of a sampling frequency, f_s,

$$\omega_{act} - \omega_c = \frac{\pm f_s}{N} \qquad \text{casma.14}$$

As an example, for a 1440 pulse per revolution shaft encoder and a sampling frequency of 100 Hz, the angular velocity measurement could be off by as much as 4.17 rpm. Clearly, if we want to use this method, we should use an encoder with as many pulses per revolution as possible and sample as infrequently as possible.

Fortunately, there is a better method commonly referred to as a 'period-based measurement'. This measurement uses both the encoder pulses and the clock to make a more accurate measurement. It is very similar to the CASMA method described previously, but instead of measuring the time between encoder pulses that are some fixed angle apart, the measurement is made between the encoder pulses that occur just after the timer events as shown below.



Our previous formula for CASMA angular velocity resolution also applies here once we change the ϕ_W to ϕ_t.

$$\left|\omega_{act} - \omega_c\right| \le \frac{\omega_{act}^2}{\phi_t f_c} \qquad \text{casma.15}$$

Once we recognize that

$$\phi_t \cong \omega_{act} \Delta t = \frac{\omega_{act}}{f_s} \qquad \text{casma.16}$$

We can show that

$$\left|\omega_{act} - \omega_c\right| \le \omega_{act} \frac{f_s}{f_c} \qquad \text{casma.17}$$

With a 40 MHz clock and a 100 Hz sample rate at 2500 rpm, the worst-case resolution would be a very acceptable 0.00625 rpm.

There is one variation on the 'period-based measurement' that should be discussed. If the timer events that drive the process cannot be sequenced with the processes that consume the data such as the data transfer process, you run the risk of excessive or variable latency. One somewhat involved method that has been used to eliminate this risk is to keep track of the number of encoder pulses that would be expected in the sample window and then basing the period measurement on fewer pulses. If done properly, this would allow just enough time for the calculation to be completed so the result is already available when the timer pulse hits. The downside to this method is that not every encoder pulse is used, so any averaging method that relies on totalizing the individual measurements will be slightly in error.

CASMA Angular Acceleration

The angular acceleration, α, over some time interval Δt, may be approximated by dividing the change in measured velocity by the time interval.

$$\alpha = \frac{\omega_i - \omega_{i-1}}{\Delta t} \qquad \text{casma.18}$$

The question becomes which velocity measurements to use and over what time interval. For CASMA filtered acceleration for a given measurement window, ϕ_W, we choose to use the angular velocities calculated using that window. We could choose to use the values one sample window, ϕ_S, apart and the associated time interval to get the most up-to-date value. But we want to avoid the error due to the fact that the velocity speeds up and slows down within one complete cycle, which affects the time interval. So, we choose to use velocities calculated one measurement window apart and the associated time required to turn through that crank angle.

So the calculated angular acceleration becomes:

$$\alpha_c = \frac{\omega_{c,i} - \omega_{c,i-1}}{\Delta t_c} \qquad \text{casma.19}$$

The actual angular acceleration we would like to calculate is:

$$\alpha_{act} = \frac{\omega_{act,i} - \omega_{act,i-1}}{\Delta t_W} \qquad \text{casma.20}$$

So the angular acceleration error would be:

$$\alpha_{act} - \alpha_c = \frac{\omega_{act,i} - \omega_{act,i-1}}{\Delta t_W} - \frac{\omega_{c,i} - \omega_{c,i-1}}{\Delta t_c} \qquad \text{casma.21}$$

Substituting in from previous equations and adopting the nomenclature, $\Delta\omega = \omega_i - \omega_{i-1}$, we get:

$$\begin{aligned}
\alpha_{act} - \alpha_c &= \frac{\Delta\omega_{act}}{\frac{\phi_W}{\omega_{act}}} - \frac{\Delta\omega_{act} \pm \frac{2\omega_{act}^2}{\phi_W f_c}}{\frac{\phi_W}{\omega_{act}} \pm \frac{1}{f_c}} \\
&= \frac{\Delta\omega_{act}\omega_{act}}{\phi_W} - \frac{\Delta\omega_{act}\omega_{act} f_c \pm \frac{2\omega_{act}^3}{\phi_W}}{\phi_W f_c \pm \omega_{act}} \\
&= \frac{\Delta\omega_{act}\omega_{act}^2 \pm 2\omega_{act}^3}{\phi_W^2 f_c \pm \phi_W \omega_{act}}
\end{aligned} \qquad \text{casma.22}$$

If we again recognize that for clock frequencies in the megahertz range and any reasonable crank angle window, $\omega_{act} << \phi_W f_c$. And since $\omega_{act} << \phi_W f_c$, it follows that $\phi_W \omega_{act} << \phi_W^2 f_c$, so the worst-case resolution for the angular acceleration calculation is essentially:

$$\begin{aligned}
\alpha_{act} - \alpha_c &= \frac{\Delta\omega_{act}\omega_{act}^2 \pm 2\omega_{act}^3}{\phi_W^2 f_c} \\
&= \frac{\Delta\omega_{act}\omega_{act}^2}{\phi_W^2 f_c} \pm \frac{2\omega_{act}^3}{\phi_W^2 f_c} \\
&= \alpha_{act}\frac{\omega_{act}}{\phi_W f_c} \pm \frac{2\omega_{act}^3}{\phi_W^2 f_c}
\end{aligned} \qquad \text{casma.23}$$

But as we have already noted, $\omega_{act} << \phi_W f_c$, so the first term essentially disappears and we have:

$$\left|\alpha_{act} - \alpha_c\right| \le \frac{2\omega_{act}^3}{\phi_W^2 f_c} \qquad \text{casma.24}$$

If we assume a clock frequency of 40 MHz, a measurement window of 720 degrees and an angular velocity of 2500 rpm, the maximum error would be 0.0543 rpm/s.

Instantaneous Angular Acceleration

If we use the velocities we calculate from a frequency-based measurement, casma.21 becomes:

$$\alpha_{act} - \alpha_c = \frac{\omega_{act,i} - \omega_{act,i-1}}{\Delta t} - \frac{\omega_{c,i} - \omega_{c,i-1}}{\Delta t} \qquad \text{casma.25}$$

Using the nomenclature, $\Delta\omega = \omega_i - \omega_{i-1}$, and the result from casma.17, we get:

$$\begin{aligned}\alpha_{act} - \alpha_c &= \frac{\Delta\omega_{act}}{\Delta t} - \frac{\Delta\omega_{act} \pm 2\frac{f_s}{N}}{\Delta t} \\ &= \frac{\pm 2 f_s}{N\Delta t} \\ &= \pm 2\frac{f_s^2}{N}\end{aligned} \qquad \text{casma.26}$$

So

$$\left|\alpha_{act} - \alpha_c\right| \le 2\frac{f_s^2}{N} \qquad \text{casma.27}$$

At a sampling frequency of 100 Hz and a 1440 pulse per revolution shaft encoder, the maximum error could be as large as 833 rpm/s. This is clearly unacceptable.

Fortunately, we have the instantaneous period-based velocity measurement to fall back on. The calculated angular acceleration is again just the velocity difference between successive sample intervals divided by the time interval Δt_n, for the last velocity measurement.

$$\alpha_c = \frac{\omega_{c,i} - \omega_{c,i-1}}{\Delta t_c} \qquad \text{casma.28}$$

The actual angular acceleration is

$$\alpha_{act} = \frac{\omega_{act,i} - \omega_{act,i-1}}{\Delta t_E} \qquad \text{casma.29}$$

So the difference between the actual and calculated angular acceleration could be as large as:

$$\begin{aligned}
\alpha_{act} - \alpha_c &= \frac{\omega_{act,i} - \omega_{act,i-1}}{\Delta t_E} - \frac{\omega_{c,i} - \omega_{c,i-1}}{\Delta t_E \pm \dfrac{1}{f_c}} \\
&= \frac{\Delta\omega_{act}}{\Delta t_E} - \frac{\Delta\omega_{act} \pm 2\omega_{act}\dfrac{f_s}{f_c}}{\Delta t_E \pm \dfrac{1}{f_c}} \\
&= \frac{\pm\dfrac{\Delta\omega_{act}}{f_c} \pm 2\omega_{act}\dfrac{f_s}{f_c}\Delta t_E}{\Delta t_E^2 \pm \dfrac{\Delta t_E}{f_c}} \\
&= \frac{\pm\Delta\omega_{act} \pm 2\omega_{act} f_s \Delta t_E}{\Delta t_E^2 f_c \pm \Delta t_E}
\end{aligned} \qquad \text{casma.30}$$

Since $\Delta t_E \cong \frac{1}{f_s}$, we can write:

$$\alpha_{act} - \alpha_c = \frac{\pm\Delta\omega_{act} f_s^2 \pm 2\omega_{act} f_s^2}{f_c \pm f_s} \qquad \text{casma.31}$$

Since $f_s << f_c$, this simplifies to:

$$\begin{aligned}
\alpha_{act} - \alpha_c &= \frac{\pm\Delta\omega_{act} f_s^2}{f_c} \pm \frac{2\omega_{act} f_s^2}{f_c} \\
&= \frac{\pm\alpha_{act} f_s}{f_c} \pm \frac{2\omega_{act} f_s^2}{f_c} \\
&= \pm\frac{2\omega_{act} f_s^2}{f_c}
\end{aligned} \qquad \text{casma.32}$$

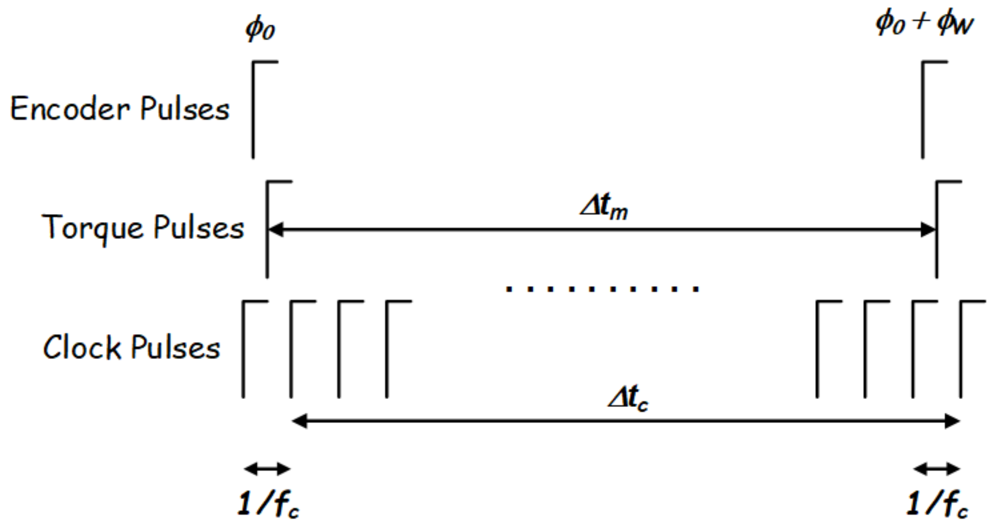
So the magnitude of the largest error would be:

$$\left|\alpha_{act} - \alpha_c\right| \le \frac{2\omega_{act} f_s^2}{f_c} \qquad \text{casma.33}$$

For 2500 rpm, a sampling frequency of 100 Hz and a clock frequency of 40 MHz the maximum error would be 1.25 rpm/s.

CASMA Torque

The torque flange outputs a frequency, f_{act}, that is proportional to torque. If we count the number of torque pulses, *m*, that occur between the encoder pulses that define the measurement window, ϕ_W, we can determine the average frequency over that interval by measuring the time, Δt_m, between the first and last torque pulses.



Because a torque flange for a motoring dynamometer needs to measure both positive and negative torque, a frequency offset, f_o, is introduced to represent zero torque. The maximum torque, $\tau_{\max}$, would occur at a frequency that is $\Delta f_{\max}$, above the frequency offset. The torque can be computed using:

$$\begin{aligned}\tau_{act} &= \frac{\tau_{\max}}{\Delta f_{\max}}\left(f_{act} - f_o\right) \\ &= \frac{\tau_{\max}}{\Delta f_{\max}}\left(\frac{m}{\Delta t_m} - f_o\right)\end{aligned} \qquad \text{casma.34}$$

We approximate Δt_m by counting the number of clock pulses that occur over a window defined by the first and last torque pulses. This measurement should be off by at most one clock pulse. The calculated torque is then:

$$\begin{aligned}\tau_c &= \frac{\tau_{\max}}{\Delta f_{\max}}\left(f_c - f_o\right) \\ &= \frac{\tau_{\max}}{\Delta f_{\max}}\left(\frac{m}{\Delta t_c} - f_o\right) \\ &= \frac{\tau_{\max}}{\Delta f_{\max}}\left(\frac{m}{\Delta t_m \pm \frac{1}{f_c}} - f_o\right)\end{aligned} \qquad \text{casma.35}$$

The worst-case resolution would be given by:

$$\begin{aligned}\tau_{act} - \tau_c &= \frac{\tau_{\max}}{\Delta f_{\max}}\left(\frac{m}{\Delta t_m} - \frac{m}{\Delta t_m \pm \frac{1}{f_c}}\right) \\ &= \frac{\tau_{\max}}{\Delta f_{\max}}\left(\frac{\pm m}{\Delta t_m^2 f_c \pm \Delta t_m}\right)\end{aligned} \qquad \text{casma.36}$$

For reasonable clock frequencies and sample window sizes, we can argue that $\Delta t_m f_c >> 1$ and therefore $\Delta t_m^2 f_c >> \Delta t_m$, so the above equation can be reduced to:

$$\begin{aligned}\tau_{act} - \tau_c &= \frac{\tau_{\max}}{\Delta f_{\max}}\left(\frac{\pm m}{\Delta t_m^2 f_c}\right) \\ &= \frac{\tau_{\max}}{\Delta f_{\max}}\left(\frac{m}{\Delta t_m}\right)\left(\frac{\pm 1}{\Delta t_m f_c}\right)\end{aligned} \qquad \text{casma.37}$$

Our original equation for torque can be rearranged to give:

$$\frac{m}{\Delta t_m} = \frac{\tau_{act}}{\tau_{\max}} \Delta f_{\max} + f_o \qquad \text{casma.38}$$

Substituting in and collecting terms gives us:

$$\begin{aligned}\left|\tau_{act} - \tau_c\right| &= \frac{\tau_{\max}}{\Delta f_{\max}}\left(\frac{\tau_{act}}{\tau_{\max}} \Delta f_{\max} + f_o\right)\left(\frac{\pm 1}{\Delta t_m f_c}\right) \\ &= \frac{1}{\Delta t_m}\left(\frac{\tau_{act}}{f_c} + \frac{\tau_{\max}}{\Delta f_{\max}} \frac{f_o}{f_c}\right)\end{aligned} \qquad \text{casma.39}$$

We can approximate Δt_m as:

$$\Delta t_m \cong \frac{\phi_W}{\omega_{act}} \quad \text{casma.40}$$

Which gives us:

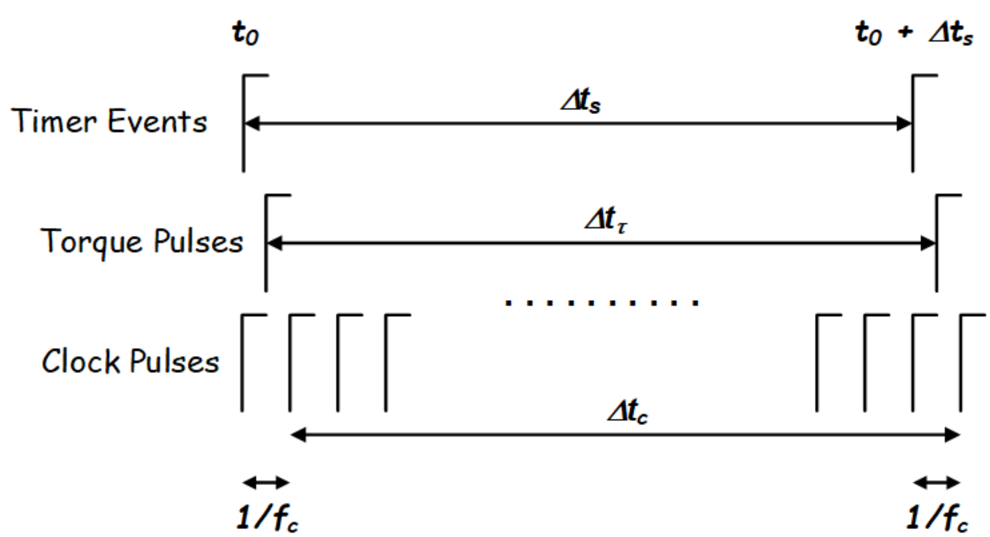
$$\left|\tau_{act} - \tau_c\right| = \frac{\omega_{act}}{\phi_W}\left(\frac{\tau_{act}}{f_c} + \frac{\tau_{\max}}{\Delta f_{\max}}\frac{f_o}{f_c}\right) \quad \text{casma.41}$$

For most transducers, the zero-frequency offset is set equal to the frequency difference corresponding to maximum torque, so the f_o and $f_{\max}$ in the second term will cancel. We can also argue that the actual torque should not exceed the maximum torque, so the worst-case resolution would occur when $\tau_{act} = \tau_{\max}$, so:

$$\left|\tau_{act} - \tau_c\right| < 2\frac{\omega_{act}}{\varphi_W}\frac{\tau_{max}}{f_c} \quad \text{casma.42}$$

Instantaneous Torque

We can measure "instantaneous" torque over some sample interval by applying the same process described above except that we now count torque pulses over a time interval defined by timer events rather than encoder pulses as shown in the diagram below.



If we count *m* torque pulses in the interval Δt_τ, the actual average torque over that interval would be:

$$\begin{aligned}\tau_{act} &= \frac{\tau_{\max}}{\Delta f_{\max}}\left(f_{act} - f_o\right) \\ &= \frac{\tau_{\max}}{\Delta f_{\max}}\left(\frac{m}{\Delta t_\tau} - f_o\right)\end{aligned}$$

casma.43

We approximate Δt_τ by counting the number of clock pulses that occur over a window defined by the first and last torque pulses. This measurement should be off by at most one clock pulse. The calculated torque is then:

$$\begin{aligned}\tau_c &= \frac{\tau_{\max}}{\Delta f_{\max}}\left(f_c - f_o\right) \\ &= \frac{\tau_{\max}}{\Delta f_{\max}}\left(\frac{m}{\Delta t_c} - f_o\right) \\ &= \frac{\tau_{\max}}{\Delta f_{\max}}\left(\frac{m}{\Delta t_\tau \pm \frac{1}{f_c}} - f_o\right)\end{aligned}$$

casma.44

The calculation of the potential error is essentially the same as for CASMA torque except that:

$$\Delta t_\tau \cong \Delta t_s = \frac{1}{f_s}$$

casma.45

Which gives us:

$$\left|\tau_{act} - \tau_c\right| = f_s\left(\frac{\tau_{act}}{f_c} + \frac{\tau_{\max}}{\Delta f_{\max}}\frac{f_o}{f_c}\right)$$

casma.46

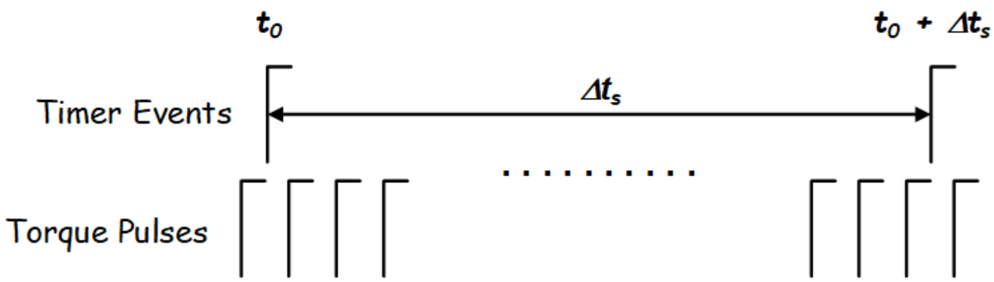
Again, for most transducers, the zero-frequency offset is set equal to the frequency difference corresponding to maximum torque, so the f_o and $f_{\max}$ in the second term will cancel. We can also argue that the actual torque should not exceed the maximum torque, so the worst-case resolution would occur when $\tau_{act} = \tau_{\max}$, so

c

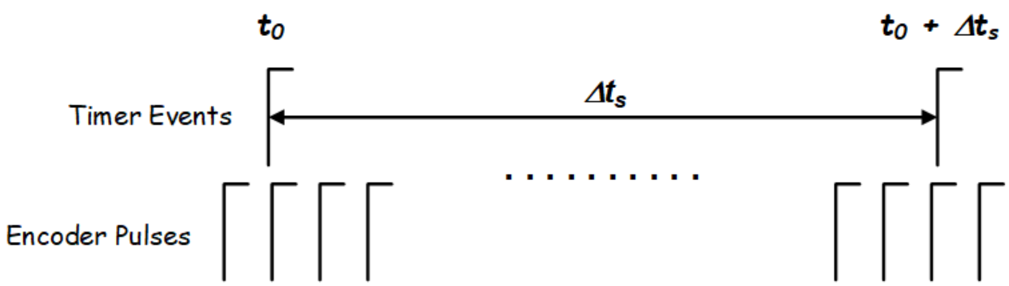
casma.47

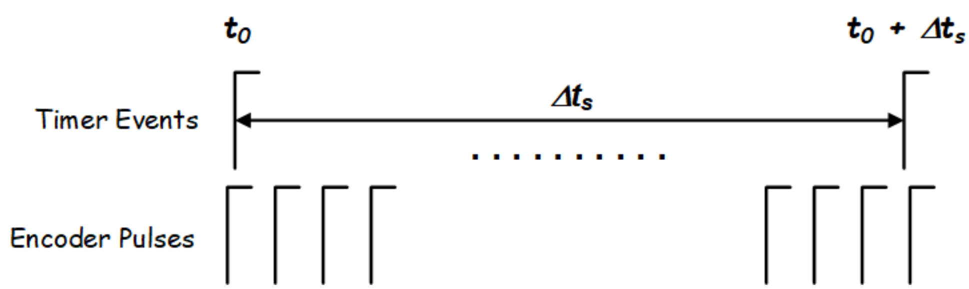
For a sampling frequency of 100 Hz and a clock frequency of 40 MHz, the maximum error would be 0.005% of maximum torque.

If we attempt to measure instantaneous torque by simply counting the number of pulses between timer events as shown below,



we would not be able to distinguish between the following two extreme cases.





If we count *m* torque pulses over the sample interval, the calculated torque would be:

$$\begin{aligned}\tau_c &= \frac{\tau_{\max}}{\Delta f_{\max}}\left(f_c - f_o\right) \\ &= \frac{\tau_{\max}}{\Delta f_{\max}}\left(\frac{m}{\Delta t_s} - f_o\right)\end{aligned} \qquad \text{casma.48}$$

But we know we could be off by almost one torque pulse in either direction, so the actual torque could be:

$$\begin{aligned}\tau_{act} &= \frac{\tau_{\max}}{\Delta f_{\max}}\left(f_\tau - f_o\right) \\ &= \frac{\tau_{\max}}{\Delta f_{\max}}\left(\frac{m \pm 1}{\Delta t_s} - f_o\right)\end{aligned} \qquad \text{casma.49}$$

So

$$\begin{aligned} \left|\tau_{act} - \tau_c\right| &\le \frac{\tau_{\max}}{\Delta f_{\max}} \frac{1}{\Delta t_s} \\ &\le \tau_{\max} \frac{f_s}{\Delta f_{\max}} \end{aligned} \qquad \text{casma.50}$$

There is obviously an advantage to using a high value for $\Delta f_{\max}$, but it is very unlikely to approach the frequency of most clocks, so this method would be inherently less accurate than directly measuring the time between torque pulses.

Example Implementation

Assume we have a counter that will be incremented every time a clock pulse is received. Also assume we have a similar counter that is incremented any time a torque pulse is received. Whenever a torque pulse is received, we will store the current value of the clock counter.

We also have an encoder input that we are counting. Assume we want to do crank angle-based sampling at a crank interval defined by *N* encoder pulses. Every N pulses we will capture the value of the clock pulse counter at that angular position, C_P. At the same time, we will capture the value of the torque pulse counter and the associated torque clock counter value, T_P and $C_{T,P}$ respectively.