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## **CyFlex® Knowledge Article**

# **Exhaust Gas Recirculation (EGR) Calculations**

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July 9, 2019



## Development

Perhaps the most common use for the equations described in the *Gas Mixture Composition* document is for calculations associated with exhaust gas recirculation (EGR). The equations may be simplified to describe a two-stream system where  $r$  moles of exhaust gas are added to one mole of fresh charge which is usually wet combustion air but could also be an air/fuel mixture. For clarity in the equations that follow, we will label the exhaust gas stream *egr*, the fresh charge stream *fresh* and the resulting mixture *charge*.

It is very difficult to directly measure EGR rate because of the high temperatures involved and the often unsteady, pulsating nature of the flow. The most common way of sidestepping this problem is to determine the value of  $r$  based on the measured and/or calculated mole fraction of a particular component gas in the fresh, egr and charge streams. Since composition measurements are often made on a dry basis, we will specifically identify water as one of the components in all of the streams. For this particular case, the generalized mixing equation for nonreacting gas streams may be rewritten as

$$\left( y_{H_2O, fresh} H_2O + y_{B, fresh} B + y_{C, fresh} C + \dots \right) + r \left( y_{H_2O, egr} H_2O + y_{B, egr} B + y_{C, egr} C + \dots \right) \quad \text{egr.1}$$

$$\Rightarrow (1+r) \left( y_{H_2O, charge} H_2O + y_{B, charge} B + y_{C, charge} C + \dots \right)$$

where:

$r$  and  $(1+r)$  are, respectively, the number of moles per unit time in the egr and charge streams per mole of the fresh stream.

$y_{X,i}$  is the mole fraction of component X in the  $i$ th stream.

If we know the wet concentration of component B in each of the streams, we can write

$$y_{B, fresh, wet} + r y_{B, egr, wet} = (1+r) y_{B, charge, wet} \quad \text{egr.2}$$

Solving for  $r$  gives

$$r = \frac{y_{B, charge, wet} - y_{B, fresh, wet}}{y_{B, egr, wet} - y_{B, charge, wet}} \quad \text{egr.3}$$

Unfortunately, most concentration measurements are made on a dry basis, which complicates the calculations considerably. To convert the mixture composition measurement from a dry to a wet basis, we use

$$y_{B, charge, wet} = y_{B, charge, dry} (1 - y_{H_2O, charge, wet}) \quad \text{egr.4}$$

But the concentration of water vapor in the charge is also a function of  $r$ , so we must use egr.2 again, but this time for water vapor

$$y_{H_2O, fresh, wet} + r y_{H_2O, egr, wet} = (1+r) y_{H_2O, charge, wet} \quad \text{egr.5}$$

Solving egr.5 for the water concentration in the charge we obtain

$$y_{H_2O,charge,wet} = \frac{y_{H_2O,fresh,wet} + r y_{H_2O,egr,wet}}{(1+r)} \tag{egr.6}$$

Substituting egr.4 and egr.6 into egr.2 gives

$$\begin{aligned} y_{B,fresh,wet} + r y_{B,egr,wet} &= (1+r) y_{B,charge,dry} (1 - y_{H_2O,charge,wet}) \\ &= (1+r) y_{B,charge,dry} \left[ 1 - \frac{y_{H_2O,fresh,wet} + r y_{H_2O,egr,wet}}{(1+r)} \right] \\ &= y_{B,charge,dry} (1+r - y_{H_2O,fresh,wet} - r y_{H_2O,egr,wet}) \end{aligned} \tag{egr.7}$$

Solving egr.7 for r gives

$$r = \frac{y_{B,charge,dry} - y_{B,fresh,wet} - y_{B,charge,dry} y_{H_2O,fresh,wet}}{y_{B,egr,wet} - y_{B,charge,dry} + y_{B,charge,dry} y_{H_2O,egr,wet}} \tag{egr.8}$$

We must also convert from a dry to a wet basis for the other concentrations using

$$y_{B,fresh,wet} = y_{B,fresh,dry} (1 - y_{H_2O,fresh,wet}) \tag{egr.9}$$

and

$$y_{B,egr,wet} = y_{B,egr,dry} (1 - y_{H_2O,egr,wet}) \tag{egr.10}$$

Substituting egr.9 and egr.10 into egr.8 gives

$$\begin{aligned} r &= \frac{y_{B,charge,dry} - y_{B,fresh,dry} (1 - y_{H_2O,fresh,wet}) - y_{B,charge,dry} y_{H_2O,fresh,wet}}{y_{B,egr,dry} (1 - y_{H_2O,egr,wet}) - y_{B,charge,dry} + y_{B,charge,dry} y_{H_2O,egr,wet}} \\ &= \frac{y_{B,charge,dry} - y_{B,fresh,dry} - y_{H_2O,fresh,wet} (y_{B,charge,dry} - y_{B,fresh,dry})}{y_{B,egr,dry} - y_{B,charge,dry} - y_{H_2O,egr,wet} (y_{B,egr,dry} - y_{B,charge,dry})} \end{aligned} \tag{egr.11}$$

And finally

$$r = \frac{(y_{B,charge,dry} - y_{B,fresh,dry}) (1 - y_{H_2O,fresh,wet})}{(y_{B,egr,dry} - y_{B,charge,dry}) (1 - y_{H_2O,egr,wet})} \tag{egr.12}$$

## Inputs

Equation egr.11 allows us to solve for  $r$  in terms of five concentrations which must either be measured directly or calculated based on other measured parameters. We will examine the possibilities for each stream in turn.

### Charge Stream Composition

To calculate  $r$ , you must measure the concentration of some component,  $B$ , of the charge. It can be a major component such as CO<sub>2</sub> or O<sub>2</sub> or some minor species such as NO<sub>x</sub> or CO. Just be sure you know the units of measurement and whether it is being made on a wet or a dry basis.

### Fresh Stream Composition

If the fresh stream is wet combustion air, the water vapor concentration may be determined from measurements of vapor and barometric pressure using

$$y_{H_2O,air,wet} = \frac{P_{vap}}{P_{bar}} \quad \text{egr.13}$$

The concentration of the selected component,  $B$ , may be measured directly or estimated based on established standards. Take care to be consistent when selecting units for concentration. For example, the mole fraction (volume fraction) of O<sub>2</sub> in standard combustion air is 0.20946, but the volume percent (which is what an O<sub>2</sub> analyzer usually reports) is 20.946 %. This gets less obvious for CO<sub>2</sub>, where the standard mass fraction is 0.00033 or 0.033 % or 330 ppm.

If you choose to use a trace species such as CO or NO<sub>x</sub>, it is generally acceptable to assume that the concentration in the combustion air is zero.

### EGR Stream Composition

Given the necessary inputs, the water vapor concentration in the exhaust can be calculated using the rigorous procedure outlined in the *Burned Gas Composition* document or, by making certain simplifying assumptions, using the method outlined in the *Exhaust Gas Composition* document. Using the latter approach and assuming that diesel fuel contains no oxygen, nitrogen, or sulfur, we have

$$y_{H_2O,exh,wet} = \frac{b + \beta/2}{a + b + \beta/4} \quad \text{egr.14}$$

Where  $a$ , the number of moles of dry air per mole of fuel carbon, is given by

$$a = \frac{A}{F}_{dry} \times \frac{(M_C + \beta M_H)}{M_{air}} \quad \text{egr.15}$$

and  $b$ , the number of moles of water per mole of fuel carbon, is given by

$$b = a \frac{P_{vap}}{P_{bar} - P_{vap}} \quad \text{egr.16}$$

and  $\beta$  is the molar hydrogen to carbon ratio for the fuel.  $M_{air}$ ,  $M_C$  and  $M_H$  are respectively the molecular weights of dry combustion air, carbon, and hydrogen.

If the fresh air to fuel ratio is not available, but a major exhaust gas component is being measured, the value of  $a$  can be back-calculated. If exhaust gas carbon dioxide concentration is being measured directly on a dry basis, then  $a$  can be calculated using

$$a = \frac{1 + y_{CO_2,exh,dry} \beta / 4}{y_{CO_2,exh,dry} - y_{CO_2,air,dry}} \quad \text{egr.17}$$

Or, if the exhaust gas oxygen concentration is being measured directly on a dry basis, the value of  $a$  can be back-calculated using

$$a = \frac{\beta / 4 (y_{O_2,exh,dry} - 1) - 1}{y_{O_2,exh,dry} - y_{O_2,air,dry}} \quad \text{egr.18}$$

The concentration of the selected component,  $B$ , must be measured directly if it is a minor species such as  $NO_x$  or  $CO$ . If it is a major exhaust gas component such as  $O_2$  or  $CO_2$ , it can either be measured directly or calculated based on the *Burned Gas Composition* or *Exhaust Gas Composition* document if the required inputs are available. Under the simplifying assumptions of the *Exhaust Gas Composition* document, the dry  $CO_2$  concentration may be calculated from

$$y_{CO_2,exh,dry} = \frac{a y_{CO_2,air,dry} + 1}{a - \beta / 4} \quad \text{egr.19}$$

Similarly, the dry  $O_2$  concentration may be calculated from

$$y_{O_2,exh,dry} = \frac{a y_{O_2,air,dry} - \beta / 4 - 1}{a - \beta / 4} \quad \text{egr.20}$$

## Final Results

Our original motivation for calculating  $r$  was to determine the composition of the intake charge. This is often expressed as the mass fraction of exhaust gas in the intake charge or  $EGR$ .

$$EGR = \left( \frac{r M_{egr}}{M_{fresh,wet} + r M_{egr}} \right) \quad \text{egr.21}$$

The oxygen concentration of the intake charge is another important parameter which may be obtained by combining and rearranging equations egr.2, egr.4 and egr.12 to give

$$y_{O_2,charge,wet} = \frac{y_{O_2,fresh,wet} + r y_{O_2,egr,wet}}{1 + r} \quad \text{egr.22}$$

In the preceding equations,  $M_{egr}$  is the molecular weight of the exhaust gas and  $M_{fresh,wet}$  is the molecular weight of the fresh stream. If the fresh stream is wet combustion air, the molecular weight is given by

$$M_{air,wet} = \frac{P_{vap}}{P_{bar}} M_{H_2O} + \frac{P_{bar} - P_{vap}}{P_{bar}} M_{air} \quad \text{egr.23}$$

The molecular weight of the exhaust gas may be calculated using the following equations from the *Exhaust Gas Composition* document

$$y_{H_2O,exh,wet} = \frac{\beta / 2 + b}{a + b + \beta / 4} \quad \text{egr.24}$$

$$y_{CO_2,exh,wet} = \frac{a y_{CO_2,air,dry} + 1}{a + b + \beta / 4} \quad \text{egr.25}$$

$$y_{O_2,exh,wet} = \frac{a y_{O_2,air,dry} - \beta / 4 - 1}{a + b + \beta / 4} \quad \text{egr.26}$$

$$y_{N_2,exh,wet} = \frac{a y_{N_2,air,dry}}{a + b + \beta / 4} \quad \text{egr.27}$$

$$y_{A,exh,wet} = \frac{a y_{A,air,dry}}{a + b + \beta / 4} \quad \text{egr.28}$$

$$M_{egr} = y_{CO_2,exh,wet} M_{CO_2} + y_{H_2O,exh,wet} M_{H_2O} + y_{O_2,exh,wet} M_{O_2} + y_{N_2,exh,wet} M_{N_2} + y_{A,exh,wet} M_A \quad \text{egr.29}$$

## Common Simplifying Assumptions

Back in the days of slide rules, a number of simplifying assumptions were traditionally made when calculating both  $r$  and  $EGR$ . There is really no excuse for continuing to use these assumptions given the computing power we now have at our fingertips, but they are presented here as a reference and a bridge to past methods.

Many of the most painful algebraic aspects of the development of the equation for  $r$  can be avoided if, at some point, we assume that the water vapor concentration in the exhaust gas and the combustion air is negligible. The equation for  $r$  becomes

$$r = \frac{(y_{B,mix,dry} - y_{B,air,dry})(1 - y_{H_2O,air,wet})}{(y_{B,exh,dry} - y_{B,mix,dry})(1 - y_{H_2O,exh,wet})} \cong \frac{(y_{B,mix,dry} - y_{B,air,dry})}{(y_{B,exh,dry} - y_{B,mix,dry})} \quad \text{egr.30}$$

We can also simplify the equation for %EGR by assuming that the exhaust gas molecular weight is equal to the combustion air molecular weight so

$$EGR = \frac{r M_{exh}}{M_{air} + r M_{exh}} \cong \frac{r}{1 + r} \quad \text{egr.31}$$

Combining egr.29 and egr.30 gives

$$EGR \cong \frac{(y_{B,mix,dry} - y_{B,air,dry})}{(y_{B,exh,dry} - y_{B,air,dry})} \quad \text{egr.32}$$

If we choose to use CO<sub>2</sub> as component B and calculate the exhaust gas concentration from measured air to fuel ratio, we can simplify egr.18 by assuming that the combustion air CO<sub>2</sub> concentration is negligible to get

$$y_{CO_2,exh,dry} = \frac{a y_{CO_2,air,dry} + 1}{a - \beta/4} \cong \frac{1}{a - \beta/4} \quad \text{egr.33}$$

If we substitute egr.32 into egr.31 and again choose to neglect the combustion air CO<sub>2</sub> concentration in the denominator, we get

$$EGR \cong \frac{(y_{CO_2,mix,dry} - y_{CO_2,air,dry})}{\left(\frac{1}{a - \beta/4} - y_{CO_2,air,dry}\right)} \cong (y_{CO_2,mix,dry} - y_{CO_2,air,dry})(a - \beta/4) \quad \text{egr.34}$$

If we substitute egr.14 into egr.33 to bring the air to fuel and fuel hydrogen to carbon ratios directly into the equation, we get

$$EGR \cong (y_{CO_2,mix,dry} - y_{CO_2,air,dry}) \left( \frac{A}{F} \Big|_{dry} \times \frac{(M_C + \beta M_H)}{M_{air}} - \beta/4 \right) \quad \text{egr.35}$$



It's common practice to assume a value for the fuel hydrogen to carbon ratio and to come up with some "magic numbers" in place of  $\beta$  and the molecular weights. If we assume  $\beta$  is 1.85,  $M_{air}$  is 28.9646,  $M_C$  is 12.011 and  $M_H$  is 1.00797, then egr.34 becomes

$$EGR \cong (y_{CO_2, mix, dry} - y_{CO_2, air, dry}) \left( 0.4791 \frac{A}{F} \Big|_{dry} - 0.4625 \right) \quad \text{egr.36}$$

There are as many different magic numbers as there are assumptions used in calculating them. Another common method is to do a more complete calculation for a range of representative operating conditions and then determine the magic numbers that give the best fit to the data. You may also see the wet air to fuel ratio used instead of the dry ratio.

The downside to magic numbers is that you now need to be careful to use the correct units when performing the above calculation. As written, the  $CO_2$  concentrations must be expressed as mole fractions. If molar percentages are used, the 100 multiplier disappears.

## First Law Data Integrity Check

If enough simplifying assumptions are made, a first law analysis of the two-stream mixing process can yield a simple relationship for EGR rate that requires only relatively robust temperature measurements as inputs. This relationship can be used as a data integrity check to flag possible faulty results based on concentration measurements and the previously described analysis.

If we assume:

1. Steady flow
2. No heat transfer to the surroundings
3. Properties are uniform for all streams
4. Kinetic energy of all streams is negligible
5. No work is done on any stream
6. Potential energy is constant
7. Streams are an ideal gas
8. Streams are perfectly mixed

Then we can write the first law equation for a control volume as

$$\begin{aligned} \dot{m}_{fresh} h_{fresh, inlet} + \dot{m}_{egr} h_{egr, inlet} &= \dot{m}_{charge, outlet} h_{charge, outlet} \\ &= (\dot{m}_{fresh} + \dot{m}_{egr}) h_{charge, outlet} \end{aligned} \quad \text{egr.37}$$

Based on our assumption that all streams are ideal gases with constant properties, we can assume a constant  $c_p$  and write

$$\begin{aligned} \dot{m}_{egr} (h_{egr, inlet} - h_{charge, outlet}) &= \dot{m}_{fresh} (h_{charge, outlet} - h_{fresh, inlet}) \\ \Rightarrow \dot{m}_{egr} c_p (T_{egr, inlet} - T_{charge, outlet}) &= \dot{m}_{fresh} c_p (T_{charge, outlet} - T_{fresh, inlet}) \end{aligned} \quad \text{egr.38}$$

Solving for the EGR mass flow rate and canceling out the constant specific heat yields

$$\dot{m}_{egr} = \dot{m}_{fresh} \frac{(T_{charge,outlet} - T_{fresh,inlet})}{(T_{egr,inlet} - T_{charge,outlet})} \quad \text{egr.39}$$

By definition, the EGR mass fraction, *EGR*, is

$$EGR = \frac{\dot{m}_{egr}}{\dot{m}_{charge}} = \frac{\dot{m}_{egr}}{\dot{m}_{egr} + \dot{m}_{fresh}} \quad \text{egr.40}$$

Combining the previous two equations yields the desired final relationship

$$EGR = \frac{(T_{charge,outlet} - T_{fresh,inlet})}{(T_{egr,inlet} - T_{fresh,inlet})} \quad \text{egr.41}$$

Our assumption of uniform properties for all streams is equivalent to the assumption that gave us egr.31. Combining egr.31 and the previous equation gives

$$EGR = \frac{(T_{charge,outlet} - T_{fresh,inlet})}{(T_{egr,inlet} - T_{fresh,inlet})} = \frac{r}{1+r} \quad \text{egr.42}$$

egr.42 can be solved for *r*, the number of moles of the egr stream per mole of fresh stream

$$r = \frac{(T_{charge,outlet} - T_{fresh,inlet})}{(T_{egr,inlet} - T_{charge,outlet})} \quad \text{egr.43}$$

which can in turn be used in egr.22 to determine the charge oxygen concentration.

The charge outlet, fresh inlet and egr inlet temperatures are usually measured as `int_mnf_t`, `ac_air_ot_t` and `egrclr_ot_t` respectively.