TRP LABORATORIES

CyFlex® Knowledge Article

Laminar Flow Element (LFE) Calculations

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Viscous Laminar Incompressible Flow

For incompressible viscous flow through a passage in the absence of significant elevation changes, the head loss, h_l , is given by:

$$h_l = \frac{\Delta p}{\rho} = f\frac{L}{D}\frac{\overline{U}^2}{2} \qquad \text{lfe.1}$$

Where Δp is the pressure drop, ρ is the density, f is the friction factor, L and D are the length and diameter of the passage respectively and $\overline{U}$ is the average flow velocity. If the flow is laminar, theory tells us that:

$$f = \frac{64}{Re} \qquad \text{lfe.2}$$

The Reynolds number, Re, is given by:

$$Re = \frac{\rho\overline{U}D}{\mu} \qquad \text{lfe.3}$$

where μ is the viscosity. Combining the above equations and solving for the average flow velocity gives:

$$\overline{U} = \frac{\Delta p D^2}{32\mu L} \qquad \text{lfe.4}$$

The actual volumetric flow rate, $\dot{V}$, is given by:

$$\dot{V} = \overline{U}\frac{\pi}{4}D^2 \qquad \text{lfe.5}$$

Combining the above two equations gives

$$\dot{V} = \frac{\Delta p}{\mu}\left(\frac{D^4\pi}{128L}\right) \qquad \text{lfe.6}$$

So, if we model an LFE as a collection of passages through which the flow is expected to be laminar, we would anticipate that the volumetric flow rate at a given viscosity would be a linear function of the pressure drop and that the characteristics of the passages could be captured as a single constant. We should be able to determine the constant through a calibration process where we flow gas of know viscosity at a known rate through the LFE and measure the resulting pressure drop.

Meriam Calibration and Correlations

Unfortunately, theory and practice do not always line up since the flow through a real device with entry and exit effects is not ideal. Typical LFEs have a nearly linear constant, but a second order term will generally improve the correlation. For a calibration run at a given viscosity which we will call μ_{std}, Meriam provides B and C coefficients for a 'classic' correlation of the form:

$$\dot{V} \quad = \quad B\Delta p + C\Delta p^2 \qquad \text{lfe.7}$$

Of course, this means that the $\frac{1}{\mu_{std}}$ term has been captured in the B and C coefficients, so to use the formula at other viscosities we must multiply by the ratio $\frac{\mu_{std}}{\mu}$. The final formula becomes:

$$\dot{V} \quad = \quad \left(B\Delta p + C\Delta p^2\right)\frac{\mu_{std}}{\mu} \qquad \text{lfe.8}$$

More recently, Meriam has begun providing coefficients for two additional correlations names after their authors and referred to as the Baker Shafer and the David Todd or Universal Calibration Curve (UCC) correlations. Both have the generalized form:

$$\dot{V} \quad = \quad \sum_{k=a}^{b} \dot{V}_k = \sum_{k=a}^{b} A_k \left(\frac{\rho\Delta p}{\mu^2}\right)^k Z \qquad \text{lfe.9}$$

Where $Z \quad = \quad \frac{\Delta p}{\mu}$ for the Baker Shafer correlation and $Z \quad = \quad \frac{\mu}{\rho}$ for the UCC correlation. But this only makes sense if there is a primary term such that:

$$\dot{V} \quad = \quad A_k \left(\frac{\rho\Delta p}{\mu^2}\right)^k Z = \quad A_k \frac{\Delta p}{\mu} \qquad \text{lfe.10}$$

From this we conclude that the summation for the Baker Shafer correlation starts with *a=0* and the UCC correlation starts with *a=1*. This is borne out by the fact that the coefficients provided by Meriam for the two correlations are virtually identical.

Unfortunately, both authors chose to remove length terms from the factors $\frac{\rho \Delta p}{\mu^2}$ and $\frac{\dot{V}}{Z}$ that would have made them non-dimensional. This means that the A_k terms provided in Meriam calibration documents are specific to the sets of units used when measuring and computing the various inputs.

For consistency, Cyflex uses base SI units for all calculations. The lfe task was written assuming that Meriam would generate the A_k coefficients using the units of micropoise for viscosity, inches of water for differential pressure, pounds per cubic foot for density, and cubic feet per minute for volume flow rate. After the lfe task was already written, it was discovered that Meriam uses units of inches of water at 4C for differential pressure, which do not have a NIST supported conversion factor and thus are not supported in Cyflex. A spreadsheet is available to recalculate the coefficients to correct to inches of water at 60F.

If we use the subscript m to denote values with Meriam units and c to indicate Cyflex base SI units, we can write:

$$\dot{V}_{k,m} = A_{k,m} \left(\frac{\rho_m \Delta p_m}{{\mu_m}^2} \right)^k Z_m \qquad \text{lfe.11}$$

and

$$\dot{V}_{k,c} = A_{k,c} \left(\frac{\rho_c \Delta p_c}{{\mu_c}^2} \right)^k Z_c \qquad \text{lfe.12}$$

If we take the ratio of these two equations, we can solve develop conversion factors that will allow us to compute new coefficients that are compatible with SI base units:

$$\frac{\dot{V}_{k,c}}{\dot{V}_{k,m}} = \frac{A_{k,c}}{A_{k,m}} \left(\frac{\rho_c \Delta p_c \, {\mu_m}^2}{\rho_m \Delta p_m \, {\mu_c}^2} \right)^k \frac{Z_c}{Z_m} \qquad \text{lfe.13}$$

Solving for the new SI based coefficients gives:

$$A_{k,c} = A_{k,m} \frac{\dot{V}_{k,c}}{\dot{V}_{k,m}} \left(\frac{\rho_m}{\rho_c} \right)^k \left(\frac{\Delta p_m}{\Delta p_c} \right)^k \left(\frac{{\mu_c}^2}{{\mu_m}^2} \right)^k \frac{Z_m}{Z_c} \qquad \text{lfe.14}$$

We can then define conversion factors:

$$c_{f,\dot{V}} = \frac{\dot{V}_{k,c}}{\dot{V}_{k,m}} \quad c_{f,\rho} = \frac{\rho_c}{\rho_m} \quad c_{f,\Delta p} = \frac{\Delta p_c}{\Delta p_m}$$
$$c_{f,\mu} = \frac{\mu_c}{\mu_m} \quad c_{f,Z} = \frac{Z_c}{Z_m} \qquad \text{lfe.15}$$

We can now write:

$$A_{k,c} = A_{k,m} \frac{c_{f,\dot{V}} \left(c_{f,\mu}\right)^{2k}}{\left(c_{f,\rho}\right)^{k} \left(c_{f,\Delta p}\right)^{k} c_{f,Z}} \qquad \text{lfe.16}$$

Z is different between the two correlations. For Baker Shafer, $Z = \frac{\Delta p}{\mu}$, so:

$$c_{f,Z,BS} = \frac{Z_c}{Z_m} = \frac{\Delta p_c \, \mu_m}{\Delta p_m \, \mu_c} = \frac{c_{f,\Delta p}}{c_{f,\mu}} \qquad \text{lfe.17}$$

So:

$$A_{k,c} = A_{k,m} \frac{c_{f,\dot{V}} \left(c_{f,\mu}\right)^{2k+1}}{\left(c_{f,\rho}\right)^{k} \left(c_{f,\Delta p}\right)^{k+1}} \qquad \text{lfe.18}$$

Due to a confusing document supplied by Meriam, the above units conversion is what is actually implemented in the lfe task and referred to as UCC. This will soon be corrected.

For the UCC correlation, $Z = \frac{\mu}{\rho}$, so:

$$c_{f,Z,UCC} = \frac{Z_c}{Z_m} = \frac{\mu_c \, \rho_m}{\mu_m \, \rho_c} = \frac{c_{f,\mu}}{c_{f,\rho}} \qquad \text{lfe.19}$$

So:

$$A_{k,c,UCC} = A_{k,m} \frac{c_{f,\dot{V}} \left(c_{f,\mu}\right)^{2k-1}}{\left(c_{f,\rho}\right)^{k-1} \left(c_{f,\Delta p}\right)^{k}} \qquad \text{lfe.20}$$

Noodling On Alternate Forms

The following was written before we had the Baker Shafer and David Todd papers as reference. It is left in this document to show another possible way to fit calibration data to give an indication of how far the lfe performance is from the ideal.

The origin of the $\frac{\rho \Delta p}{\mu^2}$ term is not at all obvious and the following is just a guess. The square of the Reynolds number is given by:

$$Re^2 = \frac{\rho^2 \overline{U}^2 D^2}{\mu^2} \qquad \text{lfe.21}$$

Solving lfe.1 for the square of the average velocity gives:

$$\overline{U}^2 = \frac{\Delta p}{\rho} \frac{2D}{f\,L} \qquad \text{lfe.22}$$

Combining the two equations gives:

$$Re^2 = \frac{\rho^2 D^2}{\mu^2} \frac{\Delta p}{\rho} \frac{2D}{f\,L} = \frac{\rho \Delta p}{\mu^2} \left(\frac{2D^3}{f\,L} \right) \qquad \text{lfe.23}$$

For incompressible viscous flow through a passage in the absence of significant elevation changes, the head loss, h_l , is given by:

$$h_l = \frac{\Delta p}{\rho} = f \frac{L}{D} \frac{\overline{U}^2}{2} \qquad \text{lfe.24}$$

Where Δp is the pressure drop, ρ is the density, f is the friction factor, L and D are the length and diameter of the passage respectively and $\overline{U}$ is the average flow velocity.

Solving lfe.1 for the friction factor gives:

$$f = \frac{D}{L} \frac{2}{\overline{U}^2} \frac{\Delta p}{\rho} \qquad \text{lfe.25}$$

The actual volumetric flow rate, $\dot{V}$, is given by:

$$\dot{V} = \overline{U}\frac{\pi}{4}D^2 \qquad \text{lfe.26}$$

Combining the above equations gives:

$$f = \frac{\pi^2 D^5}{8L}\frac{\Delta p}{\rho\dot{V}^2} \propto \frac{\Delta p}{\rho\dot{V}^2} \qquad \text{lfe.27}$$

The Reynolds number, Re , is given by:

$$Re = \frac{\rho\overline{U}D}{\mu} \qquad \text{lfe.28}$$

where μ is the viscosity. Combining this with the equation for the volumetric flow rate gives:

$$Re = \frac{4}{\pi D}\frac{\rho\dot{V}}{\mu} \propto \frac{\rho\dot{V}}{\mu} \qquad \text{lfe.29}$$

If the flow is laminar, theory tells us that:

$$f = \frac{64}{Re} \qquad \text{lfe.30}$$

If we take the natural logarithm of both sides of the equation, we get

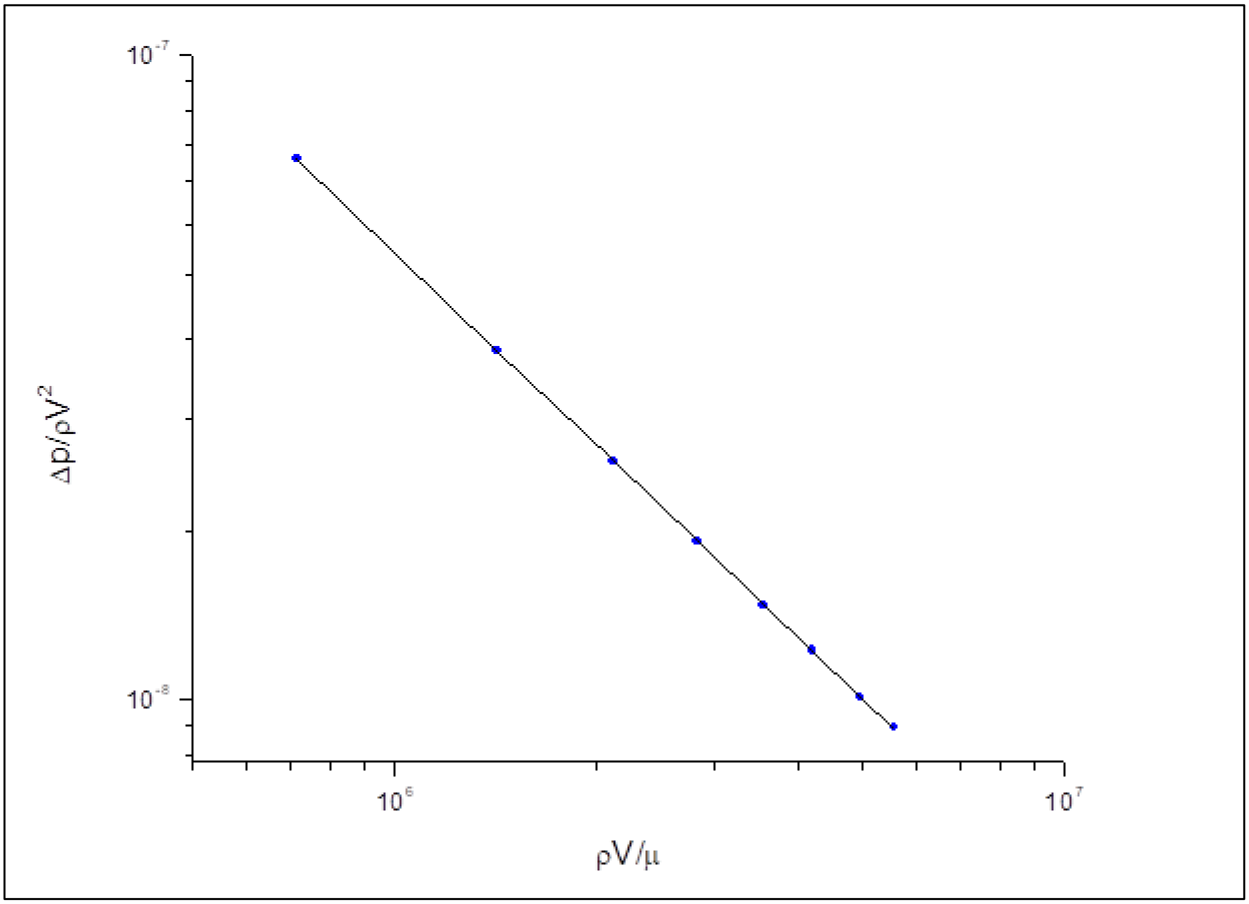
$$\ln(f) = \ln(64) - \ln(Re) \qquad \text{lfe.31}$$

So, if an LFE actually has a truly laminar characteristic, we would expect that a ln(f) vs ln(Re) plot of the calibration data would be nearly linear and have a slope close to -1.0.

If we ignore the constants and conversion factors and plot $\frac{\Delta p}{\rho\dot{V}^2}$ vs $\frac{\rho\dot{V}}{\mu}$ for the calibration data, we would hope to come up with an equation of the form:

$$\ln\left(\frac{\Delta p}{\rho\dot{V}^2}\right) = \ln(a) + b\ln\left(\frac{\rho\dot{V}}{\mu}\right) \Rightarrow \frac{\Delta p}{\rho\dot{V}^2} = a\left(\frac{\rho\dot{V}}{\mu}\right)^b \qquad \text{lfe.32}$$

where *b* would be close to -1.0.



A plot of the calibration data for a Meriam LFE model Z50MR2-8, serial number 1038000066 shows good agreement with our theory. A linear regression gives a *b* value of -0.991616.

Solving the previous equation for the volumetric flow rate gives:

$$\dot{V} = \left(\frac{\Delta p \, \mu^b}{a \, \rho^{(1+b)}} \right)^{\frac{1}{(2+b)}} \qquad \text{lfe.33}$$

As long as this form of the equation provides an adequate fit to the calibration data, we have a non-iterative closed form solution that reduces to our original theoretical equation when *b* = -1.