WHEN YOU NEED TO BE SURE

CyFlex® Knowledge Article

Pitot Tube Calculations

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Summary

To calculate the NOx load into the SCR and ultimately to calculate total mass emissions, we need to know exhaust molar flow rate. We can calculate exhaust molar flow rate if we can measure fuel and intake air ***mass*** flow rate. Because a chemical reaction takes place when the fuel is burned, unless we have the ability to perform chemical (atom) balance calculations, ***only*** mass flow measurements of fuel and air will suffice.

On test cells that do not have subsonic venturis, Servotech plans to use an averaging pitot tube to measure air flow. To accurately determine air mass flow rate, the following parameters must be measured:

- total to static pressure differential (“velocity pressure”)
- gage static pressure
- temperature
- vapor pressure
- barometric pressure

We also need an accurate measurement of the duct cross sectional area.

Of the above measurements, absolute static pressure has the potential to be the largest contributor to error in calculating mass flow rate across the entire flow range if it is not measured and assumed constant. Even when measured directly, total to static pressure differential has the potential to contribute significant error at low flow rates. Vapor pressure (via molecular weight) and temperature should be relatively minor contributors to mass flow measurement error.

Basic Calculations

According to the manufacturer, the total to static pressure differential (“velocity pressure”), P_v, in inches of water column measured by a Paragon Controls Incorporated FE-1000 insertion type air flow sensing element is given by

$$P_v = \rho\left(\frac{V}{C}\right)^2 \qquad \text{ptc.1}$$

where V is the average velocity along the probe in feet per minute, ρ is the density in pounds per cubic foot and C is a constant value of 1096.7. Solving the above equation for average velocity gives

$$V = C\sqrt{\frac{P_v}{\rho}} \qquad \text{ptc.2}$$

The manufacturer claims that the actual volumetric flow rate, Q, in cubic feet per minute is given by

$$Q = VA \qquad \text{ptc.3}$$

where A is the cross-sectional area of the duct in square feet.

Because we are ultimately interested in computing the exhaust mass flow rate, we need to know the intake air flow rate. The intake air flow rate, $\dot{m}$, is simply the actual volumetric flow rate times the density or

$$\dot{m} = \rho Q = \rho VA = \rho C\sqrt{\frac{P_v}{\rho}}A = CA\sqrt{\rho P_v} \qquad \text{ptc.4}$$

Assuming an ideal gas, the density is given by

$$\rho = \frac{PM}{R_u T} \qquad \text{ptc.5}$$

Where P is the absolute static pressure, M is the molecular weight of the gas, R_u is the universal gas constant and T is the absolute temperature. Combining the above two equations gives us

$$\dot{m} = CA\sqrt{\frac{PM}{R_u T}P_v} \qquad \text{ptc.6}$$

For moist air, the molecular weight is a function of the absolute vapor pressure, P_{vap}, and the barometric pressure, P_{bar}

$$M = \frac{P_{vap}}{P_{bar}}M_{h2o} + \frac{P_{bar} - P_{vap}}{P_{bar}}M_{dry\ air} \qquad \text{ptc.7}$$

where M_{h2o} is the molecular weight of water and $M_{dry\ air}$ is the molecular weight of dry air. If a gage pressure transducer is used to measure the static pressure, the barometric pressure has to be factored in to get absolute pressure. So

$$P = P_{gage} + P_{bar} \qquad \text{ptc.8}$$

where P_{gage} is the measured gage static pressure.

Combined Standard Uncertainty

ANSI/NCSL Z540-2-1997, the U.S. Guide to the Expression Of Uncertainty in Measurement, states that for a measurand *Y* that is not measured directly, but is determined from *N* other quantities, $X_1, X_2, \ldots, X_N$ through a functional relationship *f*:

$$Y = f\left(X_1, X_2, \ \ldots \ , X_N\right) \qquad \text{ptc.9}$$

the combined standard uncertainty $u_c(y)$ is the positive square root of the combined variance $u_c^2(y)$ which is given by

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\delta f}{\delta x_i} \right)^2 u^2(x_i)$$ ptc.10

where:

$u(x_i)$ is the standard uncertainty in the *ith* parameter used in the calculation of *Y*.

$\frac{\delta f}{\delta x_i}$ is the partial derivative, often called the sensitivity coefficient, of the measurand with respect to the *ith* parameter.

Mass Flow Uncertainty

In this case, we are interested in measuring the mass flow rate of air. So starting with the basic equation

$$\dot{m} = CA(\rho P_v)^{\frac{1}{2}}$$ ptc.11

We find

$$\frac{\delta \dot{m}}{\delta \rho} = \frac{CA}{2} \left(\frac{P_v}{\rho} \right)^{\frac{1}{2}} = \frac{CA}{2} \left(\frac{P_v R_u T}{PM} \right)^{\frac{1}{2}}$$ ptc.12

and

$$\frac{\delta \dot{m}}{\delta P_v} = \frac{CA}{2} \left(\frac{\rho}{P_v} \right)^{\frac{1}{2}}$$ ptc.13

Starting with the equation for density

$$\rho = \frac{PM}{R_u T}$$ ptc.14

We get

$$\frac{\delta \rho}{\delta P} = \frac{M}{R_u T}$$ ptc.15

$$\frac{\delta \rho}{\delta M} = \frac{P}{R_u T}$$ ptc.16

$$\frac{\delta \rho}{\delta T} = \frac{PM}{R_u T^2}$$ ptc.17

Using the chain rule

$$\frac{\delta \dot{m}}{\delta P} = \frac{\delta \dot{m}}{\delta \rho}\frac{\delta \rho}{\delta P} = \frac{CA}{2}\left(\frac{P_v R_u T}{PM}\right)^{\frac{1}{2}} \frac{M}{R_u T} = \frac{CA}{2}\left(\frac{P_v M}{P R_u T}\right)^{\frac{1}{2}} \quad \text{ptc.18}$$

$$\frac{\delta \dot{m}}{\delta M} = \frac{\delta \dot{m}}{\delta \rho}\frac{\delta \rho}{\delta M} = \frac{CA}{2}\left(\frac{P_v R_u T}{PM}\right)^{\frac{1}{2}} \frac{P}{R_u T} = \frac{CA}{2}\left(\frac{P_v P}{M R_u T}\right)^{\frac{1}{2}} \quad \text{ptc.19}$$

$$\frac{\delta \dot{m}}{\delta T} = \frac{\delta \dot{m}}{\delta \rho}\frac{\delta \rho}{\delta T} = \frac{CA}{2}\left(\frac{P_v R_u T}{PM}\right)^{\frac{1}{2}} \frac{PM}{R_u T^2} = \frac{CA}{2}\left(\frac{P_v PM}{R_u}\right)^{\frac{1}{2}} \frac{1}{T^{\frac{3}{2}}} \quad \text{ptc.20}$$

Mass Flow Uncertainty Due to Differential Pressure

If we want to calculate the percent error due to individual measured inputs, we can consider individual terms of the combined uncertainty equation. Consider the effect of uncertainty in the total to static pressure differential, ,

$$\frac{\Delta \dot{m}}{\Delta P_v} \cong \frac{\delta \dot{m}}{\delta P_v} = \frac{CA}{2}\left(\frac{\rho}{P_v}\right)^{\frac{1}{2}} \quad \text{ptc.21}$$

so

$$\Delta \dot{m} \cong \frac{CA}{2}\left(\frac{\rho}{P_v}\right)^{\frac{1}{2}} \Delta P_v \quad \text{ptc.22}$$

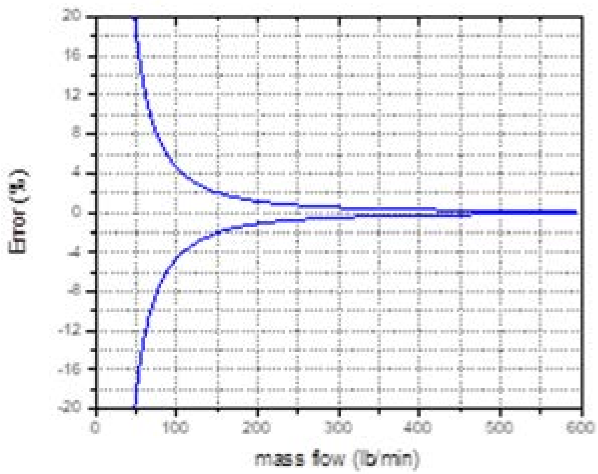
To put this in terms of a fraction of the measured mass flow

$$\frac{\Delta \dot{m}}{\dot{m}} \cong \frac{\frac{CA}{2}\left(\frac{\rho}{P_v}\right)^{\frac{1}{2}} \Delta P_v}{CA\left(\rho P_v\right)^{\frac{1}{2}}} = \frac{\Delta P_v}{2P_v} \quad \text{ptc.23}$$

The specification for one of the differential pressure transducers being used is ±0.25% of range. So in this case, the equation becomes

$$\frac{\Delta \dot{m}}{\dot{m}} \cong \frac{\pm 0.0025 \times P_{v,range}}{2P_v} \quad \text{ptc.24}$$

Plotting the % error versus mass flow for this device gives a very sobering picture. Clearly the error becomes unacceptably large well before we reach idle air flow (~30[lb/min] for a 95L). If this measurement is to be used at low flow rates, one or more additional differential pressure sensors will be needed at the low end of the range.



Mass Flow Uncertainty Due to Static Pressure

Similar formulas can be derived for the errors due to inaccuracies in the measurement of other inputs. For absolute static pressure

$$\frac{\Delta \dot{m}}{\Delta P} \cong \frac{\delta \dot{m}}{\delta P} = \frac{CA}{2}\left(\frac{P_v M}{P R_u T}\right)^{\frac{1}{2}} \qquad \text{ptc.25}$$

Which leads to

$$\frac{\Delta \dot{m}}{\dot{m}} \cong \frac{\frac{CA}{2}\left(\frac{P_v M}{P R_u T}\right)^{\frac{1}{2}} \Delta P}{CA\left(\frac{PM}{R_u T} P_v\right)^{\frac{1}{2}}} = \frac{\Delta P}{2P} \qquad \text{ptc.26}$$

The absolute static pressure could change for several reasons. The barometric pressure will regularly vary by ±2 inches of mercury due to weather conditions. For pitot tubes installed in air ducts downstream of a restrictor valve, the engine will draw a vacuum that is a function of air flow rate. The maximum design condition is 80 inches of water column vacuum.

If the mass flow calculation is done assuming a fixed absolute static pressure and we consider the minimum pressure condition, the associated error in calculated mass flow rate could be as high as 36%.

If we use reasonably accurate transducers to measure both the barometric pressure and the gage static pressure, it should be possible to realize an acceptably small error contribution due to static pressure measurement.

Mass Flow Uncertainty Due to Molecular Weight

For molecular weight

$$\frac{\Delta\dot{m}}{\Delta M} \cong \frac{\delta\dot{m}}{\delta M} = \frac{CA}{2}\left(\frac{P_v P}{MR_u T}\right)^{\frac{1}{2}} \qquad \text{ptc.27}$$

So

$$\frac{\Delta\dot{m}}{\dot{m}} \cong \frac{\frac{CA}{2}\left(\frac{P_v P}{MR_u T}\right)^{\frac{1}{2}}\Delta M}{CA\left(\frac{PM}{R_u T}P_v\right)^{\frac{1}{2}}} = \frac{\Delta M}{2M} \qquad \text{ptc.28}$$

The combustion air handler is designed to control humidity control over the range from ambient (as low as 0 grains to pound of dry air) to 256.8 grains per pound of dry air. The nominal target value is 75 grains per pound of dry air. The mass ratio to mole ratio is given by

$$\frac{m_{h_2o}}{m_{dry\,air}} = \frac{n_{h_2o}}{n_{dry\,air}}\frac{M_{h2o}}{M_{dry\,air}} \Rightarrow \frac{n_{h_2o}}{n_{dry\,air}} = \frac{m_{h_2o}}{m_{dry\,air}}\frac{M_{dry\,air}}{M_{h2o}} \qquad \text{ptc.29}$$

The molecular weight is given by

$$\begin{aligned} M &= \frac{n_{h_2o}}{n_{h_2o}+n_{dry\,air}}M_{h2o} + \frac{n_{dry\,air}}{n_{h_2o}+n_{dry\,air}}M_{dry\,air} \\ &= \frac{\frac{n_{h_2o}}{n_{dry\,air}}}{\frac{n_{h_2o}}{n_{dry\,air}}+1}M_{h2o} + \frac{1}{\frac{n_{h_2o}}{n_{dry\,air}}+1}M_{dry\,air} \end{aligned} \qquad \text{ptc.30}$$

A potential simplification would be to calculate the molecular weight under the assumption that the humidity is constant at the nominal target value of 75 grains per pound of dry air. This would result in an error in calculated mass flow of 0.65% of the nominal value at the low humidity condition and 1.5% of the actual value at the high humidity condition. These are relatively small errors, but if the vapor pressure and barometer are available, we should do the necessary calculation to eliminate this source of error.

Mass Flow Uncertainty Due to Temperature

And finally for temperature

$$\frac{\Delta \dot{m}}{\Delta T} \cong \frac{\delta \dot{m}}{\delta T} = \frac{CA}{2}\left(\frac{P_v PM}{R_u}\right)^{\frac{1}{2}} \frac{1}{T^{\frac{3}{2}}} \qquad \text{ptc.31}$$

So

$$\frac{\Delta \dot{m}}{\dot{m}} \cong \frac{\frac{CA}{2}\left(\frac{P_v PM}{R_u}\right)^{\frac{1}{2}} \frac{1}{T^{\frac{3}{2}}} \Delta T}{CA\left(\frac{PM}{R_u T} P_v\right)^{\frac{1}{2}}} = \frac{\Delta T}{2T} \qquad \text{ptc.32}$$

There is already a temperature sensor installed at the pitot tube location. Let's assume for the moment that it is a very low-grade thermocouple only capable of measuring 2 degrees centigrade. Note that the temperature in the calculation is absolute, so the amount of error introduced is only 0.33%. In cells where the air is drawn directly from the cells and the temperature is not well controlled, fluctuations could be significant and cannot be ignored. We also need to understand the effect of temperature on other sensors if they are located in the test cell environment.

Mass Flow Calculations using Integer Math

The PLC used to measure the absolute pressure, pressure differential and temperature is only capable of doing math using signed 32-bit integers. Square root calculations are also unavailable. We need to develop a method for performing the required operations without significant loss of resolutions.

Signed 32-bit integers can represent numbers as large as $2^{32} - 1$ (2,147,483,647) before they 'roll over' and represent negative numbers. So, we will get a less than meaningful result if any mathematical operation produces a number that exceeds this value.

On the other end of the spectrum, a number like 2.8852 will become simply 2 when represented as an integer. If we want to maintain 5 significant digits of resolution, we will need to multiply by 10000 and then divide by the same number later on when it will not result in a loss of resolution.

Note that all the analog inputs we will be using for our calculations come from 12-bit A/Ds, so they are only good to one part in 4095. There is no point in going to extremes to more resolution than that.

The final integer value we calculate on the PLC will be sent via a 12-bit analog output to the SEEC, so we need a value that will scale easily when divided by 4095. Since the maximum air flow rate per bank for a 95L engine is around 360 lbm/min, we somewhat arbitrarily choose 0.1 lbm/min per bit as the scaling factor.

Since we have to deal directly with the output of 12-bit A/Ds to represent our absolute pressure, temperature, and differential pressure, we need to know the relationship between the reported counts and measured engineering values. For our absolute pressure measurements, we are using an absolute pressure transducer that has a range from 600 to 1100 mbar, but it only outputs 2.5 volts at the high pressure into a 0 – 5-volt A/D. So, if *p* is the number of A/D counts, then the absolute pressure in mbar is given by

$$P[mbar] = \frac{500\,p}{2047} + 600 \qquad \text{ptc.33}$$

For differential pressure, if *x* is the number of counts, the pressure differential in inches of water column is given by

$$P_v[in\_h_2o] = \frac{0.5\,x}{4095} \qquad \text{ptc.34}$$

And for temperature in Rankin, if *t* is the number of counts

$$T[R] = \frac{t}{10} + 460 \qquad \text{ptc.35}$$

We can calculate a value equal to 10 times the mass flow rate in lbm/min using

$$10\,\dot{m} = 10\,CA\sqrt{\rho P_v} \qquad \text{ptc.36}$$

where *A* has units of square feet, ρ has units of pounds per cubic foot and P_v is in inches of water column.

If we use the ideal gas constant for air and apply the necessary conversions

$$\begin{aligned}
\rho[lbm/ft3] &= \frac{P}{RT} \\
&= \frac{\left(\frac{500}{2047}p + 600\right)\left(\frac{14.50377[lbf/in2]}{1000[mbar]}\right)(144[in2/ft2])}{53.33[ft-lbf/lbm-R]\left(\frac{t}{10}+460\right)} \\
&= \frac{\left(\frac{500}{2047}p + 600\right)}{\left(\frac{t}{10}+460\right)}0.0391626266 \\
&= \frac{\left(\frac{500}{2047}p + 600\right)}{(t+4600)}0.391626266 \\
&= \frac{(10\,p + 24564)}{(t+4600)}0.00956585898
\end{aligned} \qquad \text{ptc.37}$$

The value for C is given by the manufacturer as 1096.7 and the area of a 23-inch duct is 2.8852 square feet. Plugging all the above into our mass flow rate equation gives

$$10\,\dot{m} = 10\,CA\sqrt{\rho P_v}$$

$$= (10)(1096.7)(2.8852)\sqrt{\frac{(10\,p+24564)}{(t+4600)}\left(\frac{0.5\,x}{4095}\right)0.00956585898}$$

$$= 34.1966689\sqrt{\frac{(10\,p+24564)}{(t+4600)}x} \qquad \text{ptc.38}$$

$$= \frac{34.1966689}{10\,000\,000}\sqrt{\frac{(10\,p+24564)}{(t+4600)}x}$$

By examining individual terms, we find that we lose resolution when the square root is calculated because the number becomes too small. So, we multiply the value under the square root by 10000 and then divide the term outside the square root by 100 to compensate.

$$10\,\dot{m} = \frac{341966689}{1000\,000\,000}\sqrt{\frac{(10\,p+24564)}{(t+4600)}x(10000)} \qquad \text{ptc.39}$$

It is important to control the order in which terms are calculated. The following sequence yields acceptable results across the flow range of interest:

$$Z1 = (10\,p+24564)(x) \qquad \text{ptc.40}$$

$$Z2 = \left[\frac{Z1}{(t+4600)}\right](10000) \qquad \text{ptc.41}$$

An iterative solution is required to calculate the square root. The PLC performs the calculation based on a timer and as long as the inputs are relatively stable, this calculation should produce the desired result.

$$Z3 = \frac{\left(Z3+\frac{Z2}{Z3}\right)}{2} \qquad \text{ptc.42}$$

$$10\,\dot{m} = Z4 = \frac{[(Z3)(341966689)]}{1000\,000\,000} \qquad \text{ptc.43}$$

Where Z4 is the number of counts to be sent via the analog output. Note that it is important to maintain the order of operations specified by the various levels of parentheses.