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SGS

CyFlex® Knowledge Article

Real-Time CVS Flow Equations for Subsonic Venturies

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1 Introduction

This document provides the derivation of working formulas for the calculation of flow rate in a subsonic flow venturi (SSV) with a target application of measuring flow in a CVS system. Various assumptions have been made in determining the fluid properties for a simplified real-time flow equation. Details of the derivation of the flow equation and justification of the assumptions made in the development of this equation are also described. The flow equation in ASME Fluid Meters, [i], is the basis and it is shown to be equivalent to the molar flow equation in 40CFR1065.640(c).

2 Derivation of the Flow

The primary flow element (in this case a subsonic flow nozzle or venturi SSN or SSV) in the CVS tunnel reduces the flow area in the throat, which raises the velocity and reduces the static pressure. The flow rate can be determined from the measured value of this pressure difference and from knowledge of the characteristics of the flowing fluid and the flow element.

The mass flow rate for compressible fluids through subsonic differential head meters is given by [i]* as:

$$\dot{m} = \frac{\pi d^2}{4} F_a \frac{C_d}{\sqrt{1-\beta^4}} Y \sqrt{2\rho_1(P_1 - P_2)} \quad (1)$$

Where

- d is the throat diameter;
- F_a is the coefficient of thermal expansion;
- C_d is the discharge coefficient;
- Y is the expansion factor;
- β is the diameter ratio;
- ρ is the mass density;
- P is the absolute, static pressure;
- subscript 1 is the inlet conditions; and
- and subscript 2 is the throat conditions.

The diameter ratio, β can be given as

$$\beta = d/D \quad (2)$$

Where D is the diameter of the upstream pipe

The expansion factor is given by,

$$Y = \left[r^{\frac{2}{k}} \left(\frac{k}{k-1} \right) \left(\frac{1-r^{\frac{k-1}{k}}}{1-r} \right) \left(\frac{1-\beta^4}{1-\beta^4 r^{\frac{2}{k}}} \right) \right]^{\frac{1}{2}} \quad (3)$$

Where

- r is the static pressure ratio $\equiv P_2/P_1$ (4)
- k is the isentropic exponent. This is the ratio of the relative variation in pressure to the corresponding relative variation in density under reversible adiabatic (isentropic) transformation conditions. The isentropic exponent k appears in the formulae for the expansion factor and varies with the nature of the gas and with its pressure and temperature.

The pressure ratio, r, can be expressed in terms of typical measured parameters:

ΔP, differential pressure between inlet and throat;

* Numbers in the brackets represent references given at the end.

$P_{1,g}$, gage static pressure at the inlet; and

P_B , local barometric pressure.

These terms can be combined as follows to yield a working formula for calculating r :

$$P_1 = P_{1,g} + P_B \quad (5)$$

$$P_2 = rP_1 \quad (6)$$

$$\Delta P = P_1 - P_2 \quad (7)$$

$$P_2 = P_1 - \Delta P \quad (8)$$

$$rP_1 = P_1 - \Delta P \quad (9)$$

$$r = 1 - \Delta P/P_1 = 1 - \Delta P/(P_{1,g} + P_B) \quad (10)$$

The mass density ρ_1 is given by

$$\rho_1 = \frac{P_1 M}{Z_1 R_u T_1} \quad (11)$$

where M is the molecular weight of the mixture;
 R_u is the universal gas;
 Z is the compressibility factor; and
 T_1 is the absolute temperature of the flowing gas at the inlet.

The new EPA test procedures in 40CFR1065 use molar flow rates for emissions calculations. The molar flow rate is the mass flow rate divided by the molecular weight (mass) of the mixture.

$$\dot{n} = \frac{\dot{m}}{M} \quad (12)$$

The standard volume flow rate will be calculated from the molar flow rate. The standard volume flow rate is equal to the mass flow rate divided by the density of the metered gas at standard conditions.

$$\dot{Q} = \frac{\dot{m}}{\rho_s} = \frac{\dot{n}M}{\rho_s} \quad (13)$$

where $\rho_s = \frac{P_s M}{Z_s R_u T_s}$ (14)

subscript S denotes standard conditions.

So $\dot{Q} = \dot{n} \frac{Z_s R_u T_s}{P_s}$ (15)

Combining equations. (12, 1, 3, & 11) yields the equation for the molar flow rate.

$$\dot{n} = \frac{1}{M} \frac{\pi d^2}{4} F_a \frac{C_d}{\sqrt{1-\beta^4}} \left[r^{\frac{2}{k}} \left(\frac{k}{k-1} \right) \left(\frac{1-r^{\frac{k-1}{k}}}{1-r} \right) \left(\frac{1-\beta^4}{1-\beta^4 r^{\frac{2}{k}}} \right) \right]^{\frac{1}{2}} \sqrt{2} \left(\frac{P_1 M}{Z_1 R_u T_1} \right)^{\frac{1}{2}} \sqrt{P_1 - P_2} \quad (16)$$

The last term can be expressed in terms of P1 and r using equations 10 and 7 and rearranging as follows:

$$r = 1 - \frac{\Delta P}{P_1} = 1 - \frac{P_1 - P_2}{P_1} \quad (17)$$

$$r - 1 = - \frac{P_1 - P_2}{P_1} \quad (18)$$

$$P_1 - P_2 = (1-r)P_1 \quad (19)$$

Substituting eq. 19 into 16, canceling terms and rearranging terms in the order found in 40CFR1065.640 yields,

$$\dot{n} = \frac{1}{M} \frac{\pi d^2}{4} F_a \frac{C_d}{\sqrt{1-\beta^4}} \left[r^{\frac{2}{k}} \left(\frac{k}{k-1} \right) \left(\frac{1-r^{\frac{k-1}{k}}}{1-r} \right) \left(\frac{(1-\beta^4)}{1-\beta^4 r^{\frac{2}{k}}} \right) \right]^{\frac{1}{2}} \sqrt{2} \left(\frac{P_1 M}{Z_1 R_u T_1} \right)^{\frac{1}{2}} \sqrt{(1-r)P_1} \quad (20)$$

$$\dot{n} = C_d \left[2r^{\frac{2}{k}} \left(\frac{k}{k-1} \right) \left(\frac{1-r^{\frac{k-1}{k}}}{1} \right) \left(\frac{1}{1-\beta^4 r^{\frac{2}{k}}} \right) \right]^{\frac{1}{2}} \frac{\pi d^2}{4} P_1 F_a \left(\frac{1}{Z_1 M R_u T_1} \right)^{\frac{1}{2}} \quad (21)$$

$$\dot{n} = C_d \left[\frac{2k \left(1-r^{\frac{k-1}{k}} \right)}{(k-1) \left(r^{\frac{-2}{k}} - \beta^4 \right)} \right]^{\frac{1}{2}} \frac{\frac{\pi d^2}{4} P_1}{\sqrt{Z_1 M R_u T_1}} F_a \quad (22)$$

This equation can be simplified for CVS applications by taking the following assumptions:

- 1) The deviation from the ideal gas equation of state is neglected, i.e., the compressibility factor, $Z = Z_1 = Z_s = 1.0$
- 2) The ratio of the specific heat capacities of the ideal gases is used in place of isentropic exponent, $k = \gamma$
- 3) The variations in specific heat ratio is neglected, and a constant value can be taken as $\gamma = 1.399$

- 4) Molecular weight of the dilute exhaust gas mixture is the same as the molecular weight of the dilution air.
- 5) Thermal expansion is neglected, i.e. $F_a = 1.0$

Applying assumption 2, which is appropriate for gas pressures less than 25% of the critical pressure (all engine testing applications), yields

$$\dot{n} = C_d \left[\frac{2\gamma \left(1 - r^{\frac{\gamma-1}{\gamma}}\right)}{(\gamma-1) \left(r^{\frac{-2}{\gamma}} - \beta^4\right)} \right]^{\frac{1}{2}} \frac{\frac{\pi d^2}{4} P_1}{\sqrt{Z_1 M R_u T_1}} F_a \quad (23)$$

If we let

$$C_f = \left[\frac{2\gamma \left(1 - r^{\frac{\gamma-1}{\gamma}}\right)}{(\gamma-1) \left(r^{\frac{-2}{\gamma}} - \beta^4\right)} \right]^{\frac{1}{2}} \quad (24)$$

then

$$\dot{n} = C_d C_f \frac{\frac{\pi d^2}{4} P_1}{\sqrt{Z_1 M R_u T_1}} F_a \quad (25)$$

Applying assumption 5 ($F_a = 1$), equations 24 and 25 become the same equations as those given in 40CFR1065.640(c). Those CFR equations go on to use the same molar flow equation for critical flow venturies (CFV's) by evaluating C_f at a specific pressure ratio, r_{CFV} , that results in sonic flow (Mach number = 1) at the throat. The discussion on pressure ratio limits in a latter section suggest that this is appropriate for CFVs when the temperature is measured upstream.

For real time implementation, generic working formulas are desired that can be reused for other applications. For SSV gas flow rate equations, the gas composition and properties can be treated as inputs variables. The outputs will be mass flow rate, molar flow rate, standard volume flow rate, and Reynolds number at the SSV throat. It is assumed that all input constants and variables will be in base SI units and the outputs will be in base SI units. Units conversion from or to non-base SI units will be assumed to be done by other real time routines. To enhance readability of the real time code, some derived constants will be identified that can be evaluated once at startup of the real time routine. The real time working formulas will be summarized in a subsequent section based on derivations throughout this document.

Collecting the constants in the molar flow rate equation 23 yields,

$$\dot{n} = \left[\frac{2\gamma}{(\gamma-1)Z_1R_u} \right]^{\frac{1}{2}} \frac{\pi}{4} d^2 C_d \left[\frac{\left(1 - r^{\frac{\gamma-1}{\gamma}}\right)^{\frac{1}{2}}}{\left(r^{\frac{-2}{\gamma}} - \beta^4\right)} \right]^{\frac{1}{2}} \frac{P_1}{\sqrt{MT_1}} \quad (23)$$

$$\dot{Q} = \frac{\pi d^2}{4} \sqrt{2} \left(\frac{\gamma}{\gamma-1} \right)^{\frac{1}{2}} C_d R_u^{1/2} \frac{T_s}{P_s} P_1 \left[\frac{1}{MT_1} r^{\frac{2}{\gamma}} \left(1 - r^{\frac{\gamma-1}{\gamma}}\right) \left(\frac{1}{1 - \beta^4 r^{\frac{2}{\gamma}}} \right) \right]^{\frac{1}{2}} \quad (15)$$

All constants are collected into one term, A_0 , by letting

$$A_0 = \frac{\pi}{4} \sqrt{2} \left(\frac{\gamma}{\gamma-1} \right)^{\frac{1}{2}} R_u^{1/2} \frac{T_s}{P_s} \quad (16)$$

which yields the subsonic flow equation for standard volumetric flow.

$$\dot{Q} = A_0 d^2 C_d P_1 \left[\frac{1}{MT_1} \left(r^{\frac{2}{\gamma}} - r^{\frac{\gamma+1}{\gamma}} \right) \left(\frac{1}{1 - \beta^4 r^{\frac{2}{\gamma}}} \right) \right]^{\frac{1}{2}} \quad (17)$$

This equation will be used to justify assumptions 3 & 4 in the next section.

Applying assumption 3 ($\gamma = 1.4$) yields the final working formula for calculating the volumetric flow through a subsonic nozzle/venturi.

$$\dot{Q} = A_0 d^2 C_d P_1 \left[\frac{1}{MT_1} \left(r^{1.4286} - r^{1.7143} \right) \left(\frac{1}{1 - \beta^4 r^{1.4286}} \right) \right]^{\frac{1}{2}} \quad (18)$$

The constant term, A_0 , includes the parameters γ , R_u , T_s , and P_s . The values of these parameters are taken as:

$$\gamma = 1.4 \text{ (assumption 3)}$$

$$R_u = 1545.32 \frac{\text{ft} \cdot \text{lb}_f}{\text{lb-mole} \cdot ^\circ\text{R}} \quad [i]$$

EPA standard conditions are:

$$T_s = 68 \text{ }^\circ\text{F} \text{ (527.67 }^\circ\text{R)}$$

$$P_s = 29.92 \text{ in Hg @ } 32 \text{ }^\circ\text{F}$$

CRC [ii] gives a value of R_u in SI units as

$$R_u = 8314.32 \frac{\text{N} \cdot \text{m}}{\text{kg-mole K}}$$

Converting the SI value of R_u from CRC using ASTM E380-79 units conversion yields a value of 1545.3207, which agrees with ASME's value to the same number of significant figures.

Evaluating all the constants and parameters (in English units) in eqn.(16) yields

$$A_0 = 3404.789 \left\{ \frac{\text{ft}^3}{\text{min}} \cdot \left[{}^\circ\text{R} \frac{\text{lb}_m}{\text{lb-mole}} \right]^{1/2} \cdot \frac{1}{\text{in Hg@32F}} \cdot \frac{1}{\text{in}^2} \right\} \quad (19)$$

The units selected in eqn.(19) are consistent with \dot{Q} in scfm, d in inches, P_1 in inch Hg @ 32 °F, M in $\text{lb}_m/\text{lb-mole}$, and T_1 in °R. Note that C_d , r and γ are dimensionless and unitless. Other sets of units will yield different values of A_0 .

For free standing nozzles and venturies, β is zero because D is much larger than d and P_1 is barometric pressure P_B . In this case, eqn.(18) reduces to

$$\dot{Q} = A_0 d^2 C_d P_B \left[\frac{1}{MT_1} \left(r^{1.4286} - r^{1.7143} \right) \right]^{1/2} \quad (20)$$

This is the form of the flow equation used for CVS calibration nozzles/venturies.

3 Justification of Assumptions

3.1 Compressibility is Negligible ($Z = 1.0$)

All real gases deviate by varying amounts from the equation of state for an ideal gas. It has been found convenient to account for such deviations by introducing into the ideal-gas equation a new variable factor. The most common form of the modified equation is

$$Z = \frac{P}{\rho RT} \quad (21)$$

in which Z is called the compressibility factor.

The working flow formulas in the CFR for PDP & CFV-CVS neglect compressibility. This assumption is also taken here for the SFV. In effect, the compressibility at operating conditions is assumed to be the same as at calibration conditions and is rolled into C_d .

To illustrate the magnitude of error in this assumption, the Z terms from eq. (14) were analyzed for variation for dry air under extreme temperatures. The Z terms are $Z_s/(Z_1)^{1/2}$. Any non-unity value of these terms would be rolled into C_d at time of calibration. Then, the only residual error would be the deviation of $(Z_1)^{-1/2}$ from calibration conditions.

The compressibility factor of dry air at 1 atm was calculated using Redlich-Kwong equation of state [v]. It varies from 0.99953 at 77 °F to 1.00016 at 375 °F (maximum operating temperature of an SFV-CVS system). The resulting flow error at 375 °F, assuming the calibration was done at 77°F is $[1 - (Z_{375}/Z_{77})^{1/2}]$ or -0.03%.

Thus, it can be safely assumed that the effect of compressibility can be neglected and the compressibility factor can be taken as 1.0.

3.2 Ideal Gas Expansion ($k = \gamma$)

Real gas properties deviate from ideal behavior at very high pressures. ISO [iv] states that the isentropic exponent, k , can be replaced by specific heat ratio if the gas pressure is less than 25% of the critical pressure. This assumption is easily justified in this case because the typical metering pressure of 1 atm is only 2.7% of the critical pressure of air (37.2 atm [x], p 51).

3.3 Constant Specific Heat Ratio ($\gamma = 1.4$)

To show that specific heat ratio, γ , can be treated as a constant of 1.4, the flow equation (17) was evaluated with extreme values of γ and compared to equation (18) with the assumed value of $\gamma = 1.4$. Different values of A_0 were used with eq. (17) based on corresponding values of γ . The flow differences were found to be negligible.

Two cases were analyzed. The first considered only dry air. The second added 10% water to be more representative of the worst case for dilute exhaust.

The value of γ for dry air varies from 1.4017 at 77 oF to 1.3907 at 375 oF [iii]. The specific heat ratio, γ , is furthest from the assumed value of 1.4 at 375 oF. Table 3.3A lists the flow errors for this case for different flow rates. As expected the flow error increases as r decreases. Down to r of 0.53 (almost sonic), the flow error in metering dry air caused by assuming $\gamma = 1.4$ is less than 0.23%.

Table 3.3A Flow Error for Dry Air Assuming $\gamma = 1.4$

ΔP in H ₂ O@60F	r	\dot{Q}_{Eq18} scfm ($\gamma=1.4$)	\dot{Q}_{Eq17} scfm ($\gamma=1.3907$)	% Error in \dot{Q}
1	0.997544	615.945	615.9395	-0.0009
5	0.98772	1369.97	1369.909	-0.0045
10	0.975439	1924.406	1924.233	-0.0090
50	0.877196	4062.844	4060.925	-0.0472
100	0.754391	5291.785	5286.402	-0.1017
150	0.631587	5876.126	5866.365	-0.1661
190	0.533343	6018.273	6004.573	-0.2276

Other constants input to the flow calculation are:
d=6 in, D=24 in, M=28.9644 (dry air), $C_d=0.9825$, $P_1=29.92$ in Hg@32°F, $T_1=375$ °F

The value of γ for water varies from 1.3267 at 77 °F to 1.3205 at 375 °F [vi]. Again, the specific heat ratio, γ , is furthest from the assumed value of 1.4 at 375 oF. Since γ_{H_2O} is lower than air, adding water lowers γ even further. To simulate a worst case for dilute exhaust, a mixture of 10% by mass water and dry air was analyzed. At 1 atm, 10% by mass water is equivalent to 15% by volume, 4.5 in Hg water vapor pressure, 778 gr/lbm of dry air, or 130 °F dew point. At 375 °F, the mass weighted γ value of the mixture is 1.384. Table 3.3B gives a similar lists of the flow errors for this case. Down to r of 0.53, the flow error in metering wet air caused by assuming $\gamma = 1.4$ is less than 0.4%.

It can be clearly seen that the effect of the variation in γ on the calculation of flow is negligible and γ may be treated as a constant with a value of 1.4.

Table 3.3B Flow Error for Wet Air Assuming $\gamma = 1.4$

ΔP in H ₂ O@60F	r	\dot{Q}_{Eq18} scfm ($\gamma=1.4$)	\dot{Q}_{Eq17} scfm ($\gamma=1.384$)	% Error in \dot{Q}
1	0.997544	634.3864	634.3767	-0.0015
5	0.98772	1410.987	1410.878	-0.0077
10	0.975439	1982.022	1981.715	-0.0155
50	0.877196	4184.485	4181.069	-0.0816
100	0.754391	5450.221	5440.641	-0.1758
150	0.631587	6052.057	6034.688	-0.2870
190	0.533343	6198.459	6174.089	-0.3932

Other constants input to the flow calculation are:

$d=6$ in, $D=24$ in, $M=27.3048$ (10% by mass water), $C_d=0.9825$, $P_1=29.92$ in Hg @ 32°F , $T_1=375^\circ\text{F}$

3.4 Molecular Weight Based on Dilution Air ($M_{\text{dil,exh}} = M_{\text{dil}}$)

The basic flow equation for the subsonic venturi, equation (18), includes the molecular weight of the dilute exhaust, M . The dilute exhaust is a mixture of combustion products from the engine and moist air from the dilution air. This section discusses the strategy for selecting an approximation for the molecular weight that yields acceptable accuracy. The conclusion is to use the molecular weight of the dilution air as an approximation for the molecular weight of the dilute exhaust. This approximation should also be allowed/encouraged for CFV-CVS until technical amendments are passed that require it.

3.4.1 Current CFR Practice ($M_{\text{dil,exh}} = \text{constant}$)

The basic flow equation for critical flow venturi (CFV) has the same functional dependence on the molecular weight of the dilute exhaust as the subsonic flow venturi (inversely proportional to square root of M). The working formula in the CFR for CFV-CVS rolls the molecular weight into the calibration coefficient, K_v , of 86.1319-90(d)(1). Since the calibration coefficient is a constant evaluated from the calibration data, the molecular weight used for a CFV is also a constant and is equal to the value of the molecular weight of the flowing moist air at the time of calibration. The assumption of constant molecular weight is very convenient for real time calculation of CVS flow (needed to control proportional particulate sampling), but it does cause some loss of accuracy. For most labs that do not control dilution air humidity, the magnitude of the loss is significant in comparison to the propane check tolerance of $\pm 2\%$. So, a better approximation for the molecular weight is desired.

3.4.2 Proposed Method ($M_{\text{dil,exh}} = M_{\text{dil}}$)

In practice CVS dilution ratios run 3 to 1 or higher, so the majority of the dilute exhaust is comprised of dilution air. Assuming the dilute exhaust is 100 % dilution air is a better approximation to the molecular weight compared to assuming a constant based on calibration conditions. Since humidity of dilution air is already measured for dry to wet corrections, adding dilution air molecular weight to the real time CVS flow equation adds no additional cost.

3.4.3 Theoretical $M_{\text{dil,exh}}$ based on Complete Combustion

To analyze the potential errors of assuming a constant M , and the benefits gained in using the proposed variable M based on dilution air, a very accurate value of M was needed for comparison.

Because of the minute quantities of products of incomplete combustion and NO_x , the molecular weight of the raw exhaust can very accurately be calculated by assuming the exhaust contains only products of complete combustion. Most references ignore water in the combustion air, but this model included it. The fuel was taken as a generic oxygenated hydrocarbon fuel. The molecular weight of dilution air was calculated based on the equations given in a subsequent section. The molecular weight of the dilute exhaust, $M_{\text{dil,exh}}$ was then calculated by weighting M_{dil} and M_{exh} using the dilution ratio. This very accurate theoretical calculation of molecular weight uses

1. humidity of combustion air,
2. humidity of dilution air,

3. fuel composition (H/C and O/C ratios),
4. fuel air equivalence ratio, and
5. dilution ratio.

Although the fuel composition and humidities are known, the current CVS test systems do not measure enough parameters to calculate either the fuel air equivalence ratio or dilution ratio. So, this very accurate calculation based on complete combustion cannot be used in the working real time flow equations.

3.4.4 Expected Errors

To illustrate how the use of dilution air molecular weight improves accuracy, Tables 34.B, D, E & F below bound the molecular weight and flow errors under extreme conditions for four cases. The theoretical M based on complete combustion served as the basis for comparison (i.e. it is assumed as the true value when reporting errors). Percent errors are $(100 \cdot (\text{true} - \text{measured}) / \text{true})$. Results listed as the “Current CFR Practice” assume M is based on a calibration with barometric and water vapor pressures of 29.3 and 0.5 in Hg @ 32 °F, respectively, resulting in a nominal, constant $M = 28.77756$. Different humidity conditions at calibration time would generate different errors, but the range of errors would not change. Results listed as “Proposed Method” assume M is based on the dilution air molecular weight.

Table 3.4A gives the extremes of all the input parameters to the theoretical molecular weight calculations. Humidity is varied from dry (0.0) to saturated at 86 °F (1.25 in Hg @ 32 °F). Barometric pressure has a secondary impact on humidity. The lower the barometer, the higher the absolute humidity for a given vapor pressure. The minimum barometer was taken as 29.0 in Hg @ 32 °F. The extremes in the fuel formula cover diesel, methane, and methanol. Natural gas is close to methane, propane fits between diesel & methane, and gasoline is similar to diesel. The minimum fuel air equivalence ratio, ϕ , for diesel fuel (0.25) is based on a maximum expected A/F ratio of 60. The minimum ϕ for methane & methanol are based on their lean flammability limits. The maximum ϕ is stoichiometric (1.0) for all fuels because that is the limit of applicability of the theoretical complete combustion model and in practice no engine sustains operation at ϕ higher than 1.0 (excess fuel). Dilution ratio is the ratio of dilute exhaust (total) to raw exhaust molar flows. A practical range of 3 to 10 was taken.

Table 3.4A. Extremes of Input Parameters to the Molecular Weight Calculations

Parameter	Minimum	Maximum	Units
Combustion Air Water Vapor Pressure (humidity)	0.0	1.25	In Hg @ 32F
Dilution Air Water Vapor Pressure	0.0	1.25	In Hg @32F
Barometric Pressure	29.0	29.9	In Hg @32F
Fuel H/C Ratio,	1.85	4	
Fuel O/C Ratio,	0.0	1	
Fuel-Air Equivalence Ratio (ϕ) $\phi = \left(\frac{F}{A} \right)_{\text{actual}} / \left(\frac{F}{A} \right)_{\text{stoichiometric}}$	0.25 for diesel 0.50 for methane 0.51 for methanol	1.0	

Parameter	Minimum	Maximum	Units
Dilution ratio	3	10	

To evaluate the accuracy improvement using dilution air molecular weight, the following five conditions were analyzed:

1. *Dilution Air – Uncontrolled humidity in Dilution Air*
2. *Raw Exhaust – Uncontrolled Humidity in Combustion Air*
3. *Dilute Exhaust – Uncontrolled Humidity in Combustion and Dilution Air*
4. *Dilute Exhaust – Controlled Humidity in Combustion Air Only*
5. *Dilute Exhaust – Controlled Humidity in Both Combustion and Dilution Air*

The molecular weight and flow errors were calculated for the four CVS flow conditions; 1, 3, 4 & 5 (see Tables 34.B, D, E & F). The raw exhaust analysis provided input to the three dilute conditions analyzed. No errors were calculated for the raw exhaust condition because CVS emissions measurement systems do not measure/use raw exhaust flow.

3.4.4.1 Dilution Air – Uncontrolled Humidity in Dilution Air

Table 3.4B gives the results for dilution air only. This condition is important because it estimates the errors during propane checks. The dilution air is varied from lowest (dry) to highest humidity (saturated at 86 °F at the lowest expected barometer). The maximum M occurs at lowest humidity and vice versa. Adding water, which has a lower M, lowers the mixture molecular weight. The % error in M using the nominal value varies from 0.65 to -0.99% for a total range of 1.64%. The % errors in flow is nearly half because flow is inversely proportional to the square root of the molecular weight. **Therefore, the relative error in flow for the Current CFR Practice varies from -0.32 to 0.50% for a total range of 0.82%.** This error in flow is over 1/5th the allowed tolerance for propane checks. The Proposed Method has no error when metering only dilution air, and the improved accuracy will help meet propane checks.

Table 3.4B Dilution Air Molecular Weights and Flow Errors – Uncontrolled Humidity in Dilution Air

P _{dil} in Hg@32°F	M _{dil}	Current CFR Practice		Proposed Method		Comment
		% Error in M _{dil}	% Error in Q̇	% Error in M _{dil}	% Error in Q̇	
0.0	28.9644	0.6493	-0.3231	0.0	0.0	Max. M _{dil}
0.5	28.7756	-0.0067	0.0034	0.0	0.0	
1.25	28.4925	-0.9907	0.4991	0.0	0.0	Min. M _{dil}

Other constants used in calculation: P_B=29.0 in Hg@32°F

3.4.4.2 Raw Exhaust – Uncontrolled Humidity in Combustion Air

To evaluate the extremes of M for dilute exhaust, the extremes of M for raw exhaust were determined. Table 3.5C list extreme values of M_{exh} for each fuel with the corresponding combustion air humidity. Diesel fuel produces an M_{exh} that is larger than air. So, for diesel fuels, the maximum M_{exh} occurs with least water (lowest humidity) & least excess air (highest ϕ) and vice versa (case 1 & 2). Case 1 is also the highest M_{exh} under any condition on any of the fuels. Methane and methanol both produce exhaust that has an M less than air. So, their maximums occur with least water (lowest humidity) and greatest excess air (lowest ϕ) and vice versa (cases 3-6). The lowest M_{exh} under any condition is the minimum for methanol (case 6).

Table 3.4C Molecular Weight of Raw Exhaust for Different Fuels and Uncontrolled Humidity in Combustion Air

Case	Fuel	H/C Ratio	O/C Ratio	ϕ	P_B in Hg@32°F	P_{comb} in Hg@32°F	M_{exh}	Comment
1	Diesel	1.85	0	1	29.0	0.0	29.0288	Max for diesel & Maximum M_{exh} for all fuels
2	Diesel	1.85	0	0.25	29.0	1.25	28.5160	Min for diesel fuel
3	Methane	4	0	0.50	29.0	0.0	28.3214	Max for methane
4	Methane	4	0	1	29.0	1.25	27.3584	Min for methane
5	Methanol	4	1	0.51	29.0	0.0	28.2306	Max for methanol
6	Methanol	4	1	1	29.0	1.25	27.3017	Min for methanol & Minimum M_{exh} for all fuels

3.4.4.3 Dilute Exhaust – Uncontrolled Humidity in Combustion and Dilution Air

Worst case errors for uncontrolled humidity were evaluated for both the Current CFR Practice and the Proposed Method and are listed in Table 3.4D. Such conditions represent the worst operating cases of CVS systems. The table first lists the mixing ratios and humidities for each fuel that generate extreme values of $M_{dil,exh}$ which correspond to worst case errors for the Current CFR Practice – cases 1-6. The table then list the conditions for all fuels that generate extreme values in the difference between M_{exh} & M_{dil} which correspond to the worst-case errors for the Proposed Method – cases 7 & 8. The upper half of the table lists the various molecular weights for each case and the lower half reports the corresponding errors.

The range of flow errors for the Current CFR Practice is 1.56% ranging from dry air with diesel fuel to saturated air with methanol. For the same conditions, the Proposed Method reduces this error to 0.74%. **The worst-case range of flow errors for the Proposed Method is 1.28%** which corresponds to a very unlikely conditions of the combustion and dilution air streams having opposite extremes in humidity (i.e. one dry and the other saturated).

The strategy for finding extreme values in $M_{dil,exh}$ was to identify extreme values of the components (M_{exh} & M_{dil}) and select the dilution ratio limit that favored the worst-case component (i.e. the highest/lowest mixture value occurs for the case of highest/lowest component values with the mixture ratio limit favoring the higher/lower component).

For diesel fuel, the maximum $M_{dil,exh}$ (case 1) is found by noting that its maximum M_{exh} is larger than the maximum M_{dil} , so the maximum $M_{dil,exh}$ occurs with highest M_{exh} & M_{dil} and lowest dilution ratio. The reverse strategy is used to find the minimum $M_{dil,exh}$ (case 2). The minimum M_{exh} is larger than the minimum M_{dil} , so the minimum $M_{dil,exh}$ occurs with lowest M_{exh} & M_{dil} and highest dilution ratio. The same approach was used for the other fuels list as cases 3-6.

The strategy for finding extreme values in the difference in the molecular weights between dilution air and dilute exhaust (cases 7 & 8) was to simply match the maximum M_{exh} with the minimum M_{dil} and vice versa at the lowest dilution ratio.

Table 3.4D Dilute Exhaust Molecular Weights and Flow Errors – Uncontrolled Humidity in Combustion and Dilution Air.

Case	Fuel	ϕ	P_B , in Hg	P_{comb} , in Hg	P_{dil} , in Hg	Dilution Ratio	M_{exh}	M_{dil}	$M_{dil,exh}$	Comment
1	Diesel	1	29	0	0	3	29.0288	28.9644	28.9859	Max. $M_{dil,exh}$ – diesel
2	Diesel	0.25	29	1.25	1.25	10	28.5160	28.4925	28.4948	Min. $M_{dil,exh}$ – diesel
3	Methane	0.5	29	0	0	10	28.3214	28.9644	28.9001	Max. $M_{dil,exh}$ – methane
4	Methane	1	29	1.25	1.25	3	27.3584	28.4925	28.1144	Min. $M_{dil,exh}$ – methane
5	Methanol	0.51	29	0	0	10	28.2306	28.9644	28.8910	Max. $M_{dil,exh}$ – methanol
6	Methanol	1	29	1.25	1.25	3	27.3017	28.4925	28.0956	Min. $M_{dil,exh}$ – methanol
7	Diesel	1	29	0	1.25	3	29.0288	28.4925	28.6712	Max. ($M_{dil,exh} - M_{dil}$)
8	Methanol	1	29	1.25	0	3	27.3017	28.9644	28.4102	Min. ($M_{dil,exh} - M_{dil}$)

Case	Fuel	$M_{dil,exh} - M_{dil}$	Current CFR Practice		Proposed Method		Comment
			% Error in M_{dil}	% Error in \dot{Q}	% Error in M_{dil}	% Error in \dot{Q}	
1	Diesel	0.0215	0.7239	-0.3600	0.0741	-0.0370	Largest negative flow error for Current CFR Practice

Case	Fuel	$M_{dil,exh.} - M_{dil}$	Current CFR Practice		Proposed Method		Comment
			% Error in M_{dil}	% Error in \dot{Q}	% Error in M_{dil}	% Error in \dot{Q}	
2	Diesel	0.0024	-0.9825	0.4949	0.0083	-0.0041	
3	Methane	-0.0643	0.4258	-0.2122	-0.2220	0.1112	
4	Methane	-0.3780	-2.3043	1.1725	-1.3268	0.6701	
5	Methanol	-0.0734	0.3943	-0.1966	-0.2534	0.1269	
6	Methanol	-0.3969	-2.3699	1.2064	-1.3930	0.7039	Largest positive flow error for Current CFR Practice
7	Diesel	0.1788	-0.3694	0.1852	0.6275	-0.3123	Largest negative flow error for Proposed Method
8	Methanol	-0.5542	-1.2766	0.6445	-1.9134	0.9707	Largest positive flow error for Proposed Method

3.4.4.4 Dilute Exhaust – Controlled Humidity in Combustion Air Only

Most CVS laboratories control combustion air humidity to reduce measurement variability. Table.3.4E lists the worst-case errors for the condition of combustion air controlled to 0.5 in Hg@32°F. The format is the same as Table 3.4D and the same strategy was used to determine the extreme cases.

For this more common situation, **the range of flow errors for the Current CFR Practice drops slightly to 1.39% and the Proposed Method drops to 1.06%.**

Table. 3.4E Dilute Exhaust Molecular Weights and Flow Errors – Combustion Air Humidity Controlled to 0.5 inHg@32°F and Uncontrolled Dilution Air Humidity

Case	Fuel	ϕ	P_B , in Hg	$P_{com b}$, in Hg	P_{dil} , in Hg	Dilution Ratio	M_{exh}	M_{dil}	$M_{dil,exh}$	Comment
1	Diesel	1	29	0.5	0	10	28.8505	28.9644	28.9530	Max. $M_{dil,exh} - M_{dil}$ – diesel
2	Diesel	0.25	29	0.5	1.25	10	28.7952	28.4925	28.5227	Min. $M_{dil,exh} - M_{dil}$ – diesel
3	Methane	0.5	29	0.5	0	10	28.1524	28.9644	28.8832	Max. $M_{dil,exh} - M_{dil}$ – methane
4	Methane	1	29	0.5	1.25	3	27.5873	28.4925	28.1907	Min. $M_{dil,exh} - M_{dil}$ – methane
5	Methanol	0.51	29	0.5	0	10	28.0712	28.9644	28.8751	Max. $M_{dil,exh} - M_{dil}$ – methanol
6	Methanol	1	29	0.5	1.25	3	27.5099	28.4925	28.1649	Min. $M_{dil,exh} - M_{dil}$ – methanol
7	Diesel	1	29	0.5	1.25	3	28.8505	28.4925	28.6118	Max. $(M_{dil,exh} - M_{dil})$
8	Methanol	1	29	0.5	0	3	27.5099	28.9644	28.4796	Min. $(M_{dil,exh} - M_{dil})$

Case	Fuel	M _{dil,exh} - M _{dil}	Current CFR Practice		Proposed Method		Comment
			% Error in M _{dil}	% Error in Q̇	% Error in M _{dil}	% Error in Q̇	
1	Diesel	-0.0114	0.6097	-0.3035	-0.0393	0.0197	Largest negative flow error for Current CFR Practice
2	Diesel	0.0303	-0.8855	0.4457	0.1063	-0.0531	
3	Methane	-0.0812	0.3671	-0.1831	-0.2803	0.1405	
4	Methane	-0.3017	-2.0391	1.0354	-1.0589	0.5337	
5	Methanol	-0.0893	0.3389	-0.1690	-0.3084	0.1546	
6	Methanol	-0.3275	-2.1288	1.0817	-1.1495	0.5798	Largest positive flow error for Current CFR Practice
7	Diesel	0.1193	-0.5760	0.2892	0.4189	-0.2088	Largest negative flow error for Propose Method
8	Methanol	-0.4848	-1.0355	0.5218	-1.6739	0.8476	Largest positive flow error for Proposed Method

3.4.4.5 Dilute Exhaust – Controlled Humidity in Both Combustion and Dilution Air

A few CVS laboratories control both combustion and dilution air humidity. Table.3.4F lists the worst-case errors for the condition of combustion and dilution air controlled to 0.5 in Hg@32°F. The format is the same as Table 3.4D and the same strategy was used to determine the extreme cases.

For this less common situation, **the range of flow errors for both the Current CFR Practice and the Proposed Method drop to 0.78%**. Both have the same error range because they both use a constant M in the flow equations.

Table 3.4F Dilute Exhaust Molecular Weights and Flow Errors – Combustion and Dilution Air Humidity Controlled to 0.5 in Hg@32°F

Fuel	φ	P _B in Hg@32 °F	P _{comb} in Hg@32° F	P _{dil} In Hg@32° F	Dilution Ratio	M _{exh}	M _{dil}	M _{dil,exh}	Comment
Diesel	1	29	0.5	0.5	3	28.8505	28.7756	28.8006	
Diesel	0.2 5	29	0.5	0.5	10	28.7952	28.7756	28.7776	
Methane	0.5	29	0.5	0.5	10	28.1524	28.7756	28.7133	
Methane	1	29	0.5	0.5	3	27.5873	28.7756	28.3795	
Methanol	0.5 1	29	0.5	0.5	10	28.0712	28.7756	28.7052	
Methanol	1	29	0.5	0.5	3	27.5099	28.7756	28.3537	

Fuel	ΔM	Current CFR Practice			Proposed Method		Comment
		M _{dil,exh}	R.E. M _{dil,exh}	R.E. in Flow	R.E. M _{dil,exh}	R.E. in Flow	
Diesel	0.0250	28.7776	0.0801	-0.0400	0.0868	-0.0434	
Diesel	0.0020	28.7776	0.0001	-0.0001	0.0068	-0.0034	
Methane	-0.0623	28.7776	-0.2233	0.1118	-0.2166	0.1085	
Methane	-0.3961	28.7776	-1.3831	0.6988	-1.3765	0.6954	
Methanol	-0.0704	28.7776	-0.2515	0.1260	-0.2448	0.1226	

		Current CFR Practice			Proposed Method		
Methanol	-0.4219	28.7776	-1.4728	0.7446	-1.4662	0.7413	

From the above analysis it can be seen that calculating molecular weight of the dilute exhaust in real time from molecular weight of the dilution air will improve the accuracy of the flow calculation in a SFV-CVS system. Since current systems already measure dilution air humidity there will be no additional cost in calculating the molecular weight of the dilution air. The next section describes the method of calculating molecular weight of the dilution air based on the partial pressure of water vapor of the dilution air.

3.5 Thermal Expansion is Negligible (F_a = 1.0)

If a venturi is used at a temperature other than that at which it was calibrated, the temperature difference will cause the area of the venturi throat to change. This effect is neglected in current practice of the CFR for other CVS system (CFV & PDP). Hence, it is proposed that the effect of thermal expansion should be neglected for SFV-CVS system as well.

Comparative analysis to thermal expansion equations for orifice plate shows no more than 0.6% variation in flow calculations. Thermal expansion correction factor for an orifice plate is given by [iii]:

$$F_a = 1 + \frac{2}{1 - \beta^4} (\alpha_{PE} - \beta^4 \alpha_{pipe})(t - t_{meas}) \tag{22}$$

- where, α is the coefficient of the thermal expansion
- t_{meas} temperature at the calibration conditions
- t temperature at the operating conditions
- PE, pipe orifice plate and pipe respectively

Assuming $\alpha_{PE} = 9.6 \times 10^{-6} \text{ } ^\circ\text{F}^{-1}$ and $\alpha_{pipe} = 6.7 \times 10^{-6} \text{ } ^\circ\text{F}^{-1}$ for carbon steel, $t_{meas} = 77^\circ\text{F}$ and $t = 375^\circ\text{F}$, F_a value obtained is 1.005728. The variation in flow calculation in such case is approximately 0.6% from the assumed value of $F_a = 1.0$.

Table.3.5 Effect of thermal expansion factor on flow calculation

DeltaP in H2O @ 60F	P1 inch Hg@32F	Pw inch Hg@32F	M air	r	Mmix	Q ₁ scfm	Q ₂ scfm	Percent Error in Q
10	29.3	0.5	28.9644	0.974919	28.78143	1909.857	1920.798	0.5728

Normal operating conditions at the SFV-CVS will be less severe than 375°F and the error introduced with the assumption of $F_a = 1.0$ will be even smaller. Hence CFR assumption of neglecting thermal expansion is reasonable.

4 Calculation of Molecular Weight of the Moist Air in CVS Tunnel

The molecular weight of the moist air is given by,

$$M = \chi_A M_A + \chi_W M_W \quad (23)$$

where χ is the mole fraction

subscript A refers to dry air

and subscript W refers to water vapor

Ideal gas behavior is assumed so that the mole fraction is given by

$$\chi_i = \frac{P_i}{P} \quad (24)$$

where P_i is the partial pressure of the i th component

P is the mixture absolute pressure

substituting eqn.(24) into eqn.(23) yields

$$M = \frac{P_A}{P} M_A + \frac{P_W}{P} M_W \quad (25)$$

The partial pressures in eqn.(25) must be determined at the same mixture pressure, P . Generally, the mixture pressure is measured directly and it is desirable to eliminate the partial pressure of dry air by substituting the following equation into eqn.(25)

$$P_A = P - P_W \quad (26)$$

substituting eqn.(26) in eqn.(25) yields

$$M = \frac{(P - P_W)M_A + P_W M_W}{P} \quad (27)$$

Eqn.(27) may be rearranged for the real time calculation as follows:

$$M = M_A + (M_W - M_A) \frac{P_W}{P} \quad (28)$$

The most convenient absolute mixture pressure to use in eqn.(28) is atmospheric or local barometric pressure, P_B . The partial pressure of water vapor at atmospheric pressure can be determined by a variety of methods. One common method is a dew point sensor vented to atmosphere. The molecular weights of dry air and water are [vii]:

$$M_A = 28.9644 \text{ lbm/lb-mole}$$

$$M_W = 18.0153 \text{ lb}_m/\text{lb-mole}$$

Substituting these values into eqn.(28) and using barometric pressure as the mixture pressure yields the final form of the equation for the real time calculations

$$M = 28.9644 - 10.9491 \frac{P_W}{P_B} \quad (29)$$

5 Calculation of Discharge Coefficient based on Reynolds Number

The actual rate of flow through an SFV is less than the rate of flow indicated by the theoretical flow equation used. Hence, to obtain the actual flow from the theoretical equation, an additional factor, called the “discharge coefficient, C_d ” must be introduced. This can be defined as

$$C_d = \frac{\text{actual rate of flow}}{\text{Theoretical rate of flow}}$$

The discharge coefficient C_d is either treated as a constant or is correlated to the throat Reynolds number. There are many factors involved in the installation of a SFV and the operating range of the Reynolds number can affect the functional relationship of C_d and Re .

From the initial calibration of the SFV, a look up table of C_d as a function of Re can be established. Utilizing this look up table, C_d can be calculated as a function of Re for real-time flow equation.

Another approach is to assume the form of C_d vs. Re function & use least squares to get a best fit to the calibration data. Valid range of C_d is limited by ranges of Re from calibration data.

5.1 Reynolds Number Calculation

The Reynolds number can be defined as the ratio of the inertia force to the viscous force and can be given by

$$Re = \frac{\rho V d}{\mu} \quad (31)$$

where

- ρ is the mass density
- V is the mean velocity
- d is the throat diameter
- μ is the absolute or dynamic viscosity

The Reynolds number can be related to the mass flow rate as follows:

$$\dot{m} = \rho A V \quad (32)$$

where A is the cross-sectional area of the throat.

$$\dot{m} = \rho \frac{\pi d^2}{4} V \quad (33)$$

Rearranging,

$$\frac{4\dot{m}}{\pi d} = \rho V d \quad (34)$$

Substituting eqn.(34) into eqn.(31) yields

$$Re = \frac{4\dot{m}}{\pi \mu d} \quad (35)$$

The real time form of eqn.(35) must be based on the standard volume flow rate. Again, using the assumptions of $Z_s = 1$ and combining eqns.(11b, 10 and 35) yields

$$\text{Re} = \frac{4}{\pi \mu d} \dot{Q} \frac{P_s M}{R_u T_s} \quad (36)$$

$$\text{Re} = \frac{4}{\pi} \frac{P_s}{R_u T_s} \frac{1}{d} \frac{\dot{Q} M}{\mu} \quad (37)$$

$$\text{Re} = 6.609 \cdot 10^{-4} \frac{\dot{Q} M}{d \mu} \quad (38)$$

The units in eqn.(38) are consistent with \dot{Q} in scfm, d in inches, M in lb_m/lb-mole and μ in lb_m/ft-sec.

5.2 Viscosity Calculation

The last parameter to evaluate is the viscosity. A simple formula for the viscosity of gases is given by Sutherland correlations [ix].

$$\mu = \frac{b T^{\frac{3}{2}}}{S + T} \quad (39)$$

where T is the absolute temperature & b and S are empirical constants.

The values of b and S used to compute the viscosities in the 1962 publication of "The U.S. Standard Atmosphere" from the U.S. Government Printing Office in Washington, D.C. as

$$b = 1.458 \times 10^{-6} \frac{\text{kg}}{\text{m} \cdot \text{s} \cdot \text{K}^{\frac{1}{2}}} \quad (40)$$

$$S = 110.4 \text{ K} \quad (41)$$

Converting this values into desired English unit yields

$$b = 7.3025 \times 10^{-7} \frac{\text{lb}_m}{\text{ft} \cdot \text{s} \cdot \text{R}^{\frac{1}{2}}} \quad (42)$$

$$S = 198.72 \text{ R} \quad (43)$$

Values of μ computed with eqn.(39) with constants given in eqn.(42) and eqn.(43) agree within 0.1% of tabulated values given by Bolz and Tuve [x].

5.3 Special Calculation Techniques

Since the calculation of the mass flow rate depends on the discharge coefficient, which depends on Reynolds number, which in turn depends on flow rate, a technique is used to calculate the mass flow rate with an initial guess of unity for discharge coefficient. In real time this problem is addressed by calculating the discharge coefficient based on the most recent real time value of the flow rate. To improve the convergence rate and avoid division by zero, a low limit on the flow rate is established for the discharge coefficient calculation.

6 Pressure Ratio Limit for Nozzles

Both ASME [iii] and ISO [iv] limit the pressure ratio to a minimum of 0.75 for orifices and nozzles but place no corresponding limit on venturies. The expansion factors listed for orifice meters, nozzles, and venturies are given in the ASME and ISO specifications as a function of the isentropic exponent, the static pressure ratio, and the diameter ratio (see equation 3). These equations assume that the process occurring between the pressure taps is isentropic. Under some flow conditions, the isentropic assumption is not valid. A more general theoretical expansion factor is given as follows (xii):

$$Y = \frac{C_{vc} C_{cc}}{C_{vi} C_{ci}} \left[r^{\frac{2}{k}} \left(\frac{k}{k-1} \right) \left(\frac{1-r^{\frac{k-1}{k}}}{1-r} \right) \left(\frac{1-C_{ci}^2 \beta^4}{1-C_c^2 \beta^4 r^{\frac{2}{k}}} \right) \right]^{\frac{1}{2}} \quad (44)$$

where,

C_{ci} = incompressible contraction coefficient

C_{vi} = incompressible velocity coefficient

C_{cc} = compressible contraction coefficient

C_{vc} = compressible velocity coefficient

In the case of orifice meters, the fluid is allowed to expand freely after the orifice plate, creating a significant amount of losses ($C_v > 1$). Also, the step change in pipe diameter creates a vena contracta effect because the fluid is unable to expand immediately upon entering the expansion section of the orifice meter ($C_c > 1$) [xiii]. As the pressure ratio increases, the isentropic assumption becomes less and less valid for an orifice meter. In the case of venturis, the smooth wall transitions allows the fluid to follow the contours of the meter more closely. The smooth, gradual transition of the walls to the throat of the venturi creates little contraction of the fluid beyond that created by the wall profile ($C_c \cong 1$). The expansion of the fluid after the venturi throat is constrained to the axial direction, so the losses associated with the expansion portion of the venturi are small ($C_v \cong 1$). Because of the wall boundary conditions in the venturi, the isentropic assumption is valid, and Equation 44 reduces to Equation 3 [xii].

This pressure ratio restriction is also added to neglect the difference between the upstream and downstream temperatures and allow the ASME and ISO recommended practice of measuring temperature downstream. This can be understood by considering that during the expansion process from the inlet of the venturi/nozzle to the throat, the gas pressure and temperature drop. Downstream of orifices and nozzles, the gas experiences a sudden nonisentropic expansion, where there is little pressure or temperature recovery. Therefore, this restriction should also be applied for SFNs when the temperature is measured downstream.

$$r_{\min} = 0.75, \text{ for subsonic flow nozzles with } T_1 \text{ measured downstream}$$

For venturies that have a diverging section, the gas recovers most of its pressure and temperature and no pressure ratio limit is imposed.

7 Summary of Real Time SFV-CVS Flow Equations

The following table summarizes the parameters used in the real time calculation of gas flow rate through a subsonic venturi with assumptions associated with a CVS application. Each parameter is in the listed base SI units. Conversion factors must be converted from or to other units.

Measured Quantities

Parameter	Description	Consistent Units	Source
P_B	Barometric Pressure	Pa	Barometer or absolute pressure transducer
$P_{1,g}$	Gauge Pressure at SFV inlet	Pa	Pressure transducer
ΔP	Flow Nozzle ΔP	Pa	Differential pressure transducer
T_1	Dilute exhaust temperature at SSV inlet	K	Temperature sensor
P_W	Water Vapor Pressure in dilution air	Pa	Converted from humidity sensor

Constants that may be used for dilute exhaust properties

Parameter	Description	Consistent Units	Source
$A_0 = 3404.789$	Collections of constants used in eqn.(18)	see eqn. (19)	eqn.(16)
$b = 1.458 \times 10^{-6}$	1st constant in the Sutherland viscosity correlation, eqn.(39)	kg/m-s-K ^{1/2}	eqn.(42)
$S = 110.4$	2nd constant in the viscosity correlation, eqn.(39)	K	eqn.(43)
$M_A = 28.96559$	Molecular weight (molar mass) of dry air	g/mol (a.k.a. g/g-mol, g/g-mole, kg/kg-mole)	40CFR1065.1005(f)(2)
$M_W = 18.01528$	Molecular weight (molar mass) of water	g/mol	40CFR1065.1005(f)(2)

Flow Nozzle Calibration Data

Parameter	Description	Consistent Units	Source
d	Venturi throat diameter	m	Calibration data sheet
D	Upstream pipe diameter	m	Calibration data sheet
C_d	Discharge Coefficient	unitless	Calibration data sheet with C_d as a function of Re. May be any mathematical expression such as a least squares polynomial or power series. Two forms should be supported: 1) $C_d = a_0 + a_1/\text{sqrt}(\text{Re})$ and 2) $C_d = a_0 + a_1 \text{Re} + a_2 \text{Re}^2 + \dots$

Computed Quantities

Parameter	Description	Consistent Units	Source
$M = M_w - (M_A - M_w)$	Molecular weight (molar mass) of primary dilution air, which is used for molar mass of dilute exhaust. Note, under conditions where the CVS draws air from the calibration SSV, this is the molar mass of the air entering the cal. SSV.	g/mol	eqn.(29)
$\mu = \frac{bT_1^{3/2}}{S + T_1}$	Kinematic viscosity	kg/m-s	eqn.(39)
$\text{Re} = 6.609 \cdot 10^{-4} \frac{\dot{Q}M}{d\mu}$	Reynolds number	unitless	eqn. (38)

Parameter	Description	Consistent Units	Source
$r = 1 - \frac{\Delta P}{P_1}$	Ratio of throat to inlet pressure	unitless	eqn.(6b)
$\dot{Q} = A_0 d^2 C_d P_1 \left[\frac{1}{MT_1} (r^{1.4286} - r^{1.7}) \right]$	Standard volume flow rate	scfm	eqn.(18)

where A_0 , b and S are constants given by equations (19, 42 and 43 respectively), and are independent of the flow nozzle. The input data P_B , $P_{1,g}$, P_W , ΔP and T_1 in units of in Hg @ 32 °F for the pressures and °R for the temperature. Partial pressure of water vapor is calculated using Goff-Gratch formulas [xi] which converts dew point temperature to saturation pressure. The venturi ΔP and T_1 must be converted from their measured units of in H₂O @60 °F and deg F to the required units of in Hg @ 32 °F and °R respectively. The multiplicative conversion factor for ΔP is 0.0734826. The additive factor for T_1 is 459.67.

8 References

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9 History

<i>Revision</i>	<i>Author</i>	<i>Date</i>	<i>Comments</i>
000	W.T. Martin	21Sep2004	Initial issue of document. It was based on an 11-2-2000 document titled "REAL TIME FLOW EQUATIONS FOR THE SUBSONIC FLOW VENTURI-CVS SYSTEM" by W.T. Martin, M.R.Islam, and B.A.Rankin that derived working flow equations from ASME equations with justification of simplifying assumptions. This revision derives the equations up to the form of the equations given in 40CFR1065.640.